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Domain wall from gauged $d = 4$, $\mathcal{N} = 8$ supergravity. Part II

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Abstract: The scalar potentials of the non-semi-simple $\mathrm{CSO}(p, 8-p)$ $(p = 7, 6, 5)$ gaugings of $\mathcal{N} = 8$ supergravity are studied for critical points. The $\mathrm{CSO}(7,1)$ gauging has no G_2-invariant critical points, the $\mathrm{CSO}(6,2)$ gauging has three new $\mathrm{SU}(3)$-invariant AdS critical points and the $\mathrm{CSO}(5,3)$ gauging has no $\mathrm{SO}(5)$-invariant critical points. The scalar potential of $\mathrm{CSO}(6,2)$ gauging in four dimensions we discovered provides the $\mathrm{SU}(3)$ invariant scalar potential of five dimensional $\mathrm{SO}(6)$ gauged supergravity.

The nontrivial effective scalar potential can be written in terms of the superpotential which can be read off from A_1 tensor of the theory. We discuss first-order domain wall solutions by analyzing the supergravity scalar-gravity action and using some algebraic relations in a complex eigenvalue of A_1 tensor. We examine domain wall solutions of G_2 sectors of noncompact $\mathrm{SO}(7,1)$ and $\mathrm{CSO}(7,1)$ gaugings and $\mathrm{SU}(3)$ sectors of $\mathrm{SO}(6,2)$ and $\mathrm{CSO}(6,2)$ gaugings. They share common features with each sector of compact $\mathrm{SO}(8)$ gauged $\mathcal{N} = 8$ supergravity in four dimensions.

We analyze the scalar potentials of the $\mathrm{CSO}(p, q, 8-p-q)$ gauged supergravity we have found before. The $\mathrm{CSO}(p, 6-p, 2)$ gauge theory in four dimensions can be reduced from the $\mathrm{SO}(p, 6-p)$ gauge theory in five dimensions. Moreover, the $\mathrm{SO}(p, 5-p)$ gauge theory in seven dimensions reduces to $\mathrm{CSO}(p, 5-p, 3)$ gauge theory in four dimensions. Similarly, $\mathrm{CSO}(p, q-p, 8-q)$ gauge theories in four dimensions are related to $\mathrm{SO}(p, q-p)$ $(q = 2, 3, 4, 7)$ gauge theories in other dimensions.

Keywords: M-Theory, D-branes, AdS-CFT and dS-CFT Correspondence, Supergravity Models.

Contents

1. Introduction

The domain wall (DW) and quantum field theory (QFT) correspondence is a duality between supergravity compactified on domain wall spacetimes(which are locally isometric to Anti-de Sitter (AdS) space but different from it globally) and quantum field theories describing the internal dynamics of branes that live on the boundary of such spacetimes. Compact gaugings are not the only ones for extended supergravities but there are rich structures of non-compact and non-semi-simple gaugings. These gaugings are crucial in the description of the DW/QFT correspondence as the compact gauged supergravity has played the role in the AdS/conformal field theory (CFT) duality (that is a correspondence between a certain gauged supergravities and conformal field theories).

The noncompact and non-semi-simple gauged supergravity theories could be obtained in the same way the compact SO(8) gauged supergravity theory. As a result of the complicated nonlinear tensor structure, one has to prove that the modified A_1 and A_2 tensors satisfy a rather complicated quantities to show the supersymmetry of the theory. A different method that uses known results of compact SO(8) gauged supergravity theory was found to generate noncompact and non-semi-simple gaugings such that one obtains the full nonlinear structure automatically and both gauge invariance and supersymmetry are guaranteed.

In a previous paper, Part I [1] we constructed a superpotential for known non-compact and non-semi-simple gauged supergravity theories and by looking at the energy-functional, domain wall solutions were obtained in which the role of a superpotential was very important. Moreover, by executing two successive $\mathrm{SL}(8,\mathbb{R})$ transformations on the compact gauged supergravity theory we described a T-tensor, a superpotential and domain wall solutions of non-semi-simple $\mathrm{CSO}(p,q,8-p-q)$ gaugings. One considers only scalars which are singlets of subgroup of full isometry group and is looking for critical points of the potential restricted to be a function only of the singlets. Any critical point of restricted potential is a critical point of the original full scalar potential according to Schurr's lemma [2]. In Part I, the subgroup was to be $\mathrm{SO}(p) \times \mathrm{SO}(8-p)$ for $\mathrm{SO}(p,8-p)$ and $\mathrm{CSO}(p,8-p)$ gaugings and $\mathrm{SO}(p) \times \mathrm{SO}(q) \times \mathrm{SO}(8-p-q)$ for $\mathrm{CSO}(p,q,8-p)$ gaugings.

There was an attempt [3] to study whether any critical points are present in G_2 sector for $\mathrm{SO}(7,1)$ gauging, $\mathrm{SU}(3)$ sector for $\mathrm{SO}(6,2)$ gauging and $\mathrm{SO}(5)$ sector for $\mathrm{SO}(5,3)$ gauging. Only the last one has a critical point with positive cosmological constant.

In this paper, in section 2, we examine the structure of the G_2 sector for $\mathrm{SO}(7,1)$ gauging, $\mathrm{SU}(3)$ sector for $\mathrm{SO}(6,2)$ gauging, $\mathrm{SO}(5)$ sector for $\mathrm{SO}(5,3)$ gauging and $\mathrm{SO}(3) \times \mathrm{SO}(3)$ sector for $\mathrm{SO}(4,4)$ gauging. What we are concentrating on is as follows.

- A_1 tensor and a superpotential from T-tensor for these gauged supergravity theories.

  In section 3, we repeat the procedure of section 2 for the non-semi-simple $\mathrm{CSO}(p,8-p)$ $(p=7,6,5)$ gaugings. What we are interested in is

- any critical points of G_2 sector for $\mathrm{CSO}(7,1)$ gauging, $\mathrm{SU}(3)$ sector for $\mathrm{CSO}(6,2)$ gauging and $\mathrm{SO}(5)$ sector for $\mathrm{CSO}(5,3)$ gauging.

- A_1 tensor and a superpotential from T-tensor for these non-semi-simple $\mathrm{CSO}(p,8-p)$ gauged supergravity theories.

  In section 4, we obtain domain wall solutions from direct extremization of energy-density and in order to arrive this, the observation of the presence of some algebraic relations of a superpotential will be crucial since without those relations one can not cacel out the unwanted cross terms in the energy functional. What we describe mainly is as follows.

- Domain wall solutions for non-compact $\mathrm{SO}(p,8-p)$ $(p=7,6)$ and non-semi-simple $\mathrm{CSO}(p,8-p)$ gaugings.

In section 5, we analyze the potentials of the $\mathrm{CSO}(p,q,8-p-q)$ gauged supergravity we have found in [1] before. The $\mathrm{CSO}(p,q-p,8-q)$ $(q=2,3,4,5,6,7)$ gauge theories in four dimensions are related to $\mathrm{SO}(p,q-p)$ $(q=2,3,4,5,6,7$ and $1\leq p<q)$ gauge theories in various higher dimensions. In section 6, we describe the future directions. In the appendix, we list the nonzero A_2 tensor components in the various sectors of given gauged supergravity theories.

2. The potentials of $\mathrm{SO}(p,8-p)$ gauged supergravity

We used the $\mathrm{SO}(p)\times\mathrm{SO}(8-p)$-invariant fourth rank tensor to generate transformations so that the $\mathrm{SO}(p,8-p)$ and $\mathrm{CSO}(p,8-p)$ gaugings are produced in Part I. The embedding of $\mathrm{SO}(p)\times\mathrm{SO}(8-p)$ invariant generator of $\mathrm{SL}(8,\mathbb{R})$ was such that it corresponds to the 56×56 E_7 generator which is a non-compact $\mathrm{SO}(p)\times\mathrm{SO}(8-p)$ invariant element of the $\mathrm{SL}(8,\mathbb{R})$ subalgebra of E_7. By introducing the projectors onto the corresponding eigenspaces, $\mathrm{SO}(p)\times\mathrm{SO}(8-p)$-invariant fourth rank tensor can be decomposed into these projectors. The ξ-dependent T-tensor in this case [4, 5, 6] is described by

$$\begin{aligned} T_i^{\;jkl}(\xi) = t_i^{\;jkl} - (1-\xi)\left(\overline{u}^{kl}{}_{IJ}+\overline{v}^{klIJ}\right)\times \\ \times\Bigg[\left(P_\beta^{IJKL}+\frac{1}{2}P_\gamma^{IJKL}\right)\left(u_{im}{}^{KM}\overline{u}^{jm}{}_{LM}-v_{imKM}\overline{v}^{jmLM}\right)+ \\ +P_\gamma^{IJRS}Z_{RS}^{KLMN}\left(-v_{imKL}\overline{u}^{jm}{}_{MN}+u_{im}{}^{KL}\overline{v}^{jmMN}\right)\Bigg], \end{aligned} \tag{2.1}$$

where $t_i^{\;jkl}$ in the right hand side is defined as de Wit-Nicolai T-tensor in compact $\mathrm{SO}(8)$ gauging

$$t_i^{\;jkl} = \left(\overline{u}^{kl}{}_{IJ}+\overline{v}^{klIJ}\right)\left(u_{im}{}^{JK}\overline{u}^{jm}{}_{KI}-v_{imJK}\overline{v}^{jmKL}\right)$$

and we introduce the new quantity Z_{IJKL}^{MN} in terms of quadratic projectors as follows

$$Z_{IJKL}^{MN}=\frac{1}{2}\left[\left(P_\alpha-P_\beta\right)_{IJMP}P_\gamma^{NPKL}-P_\gamma^{IJMP}\left(P_\alpha-P_\beta\right)_{NPKL}\right].$$

When $\xi=1$, the modified T-tensor reduces to t-tensor in the above. Projector $P_\alpha(P_\beta)$ projects the $\mathrm{SO}(8)$ Lie algebra onto its $\mathrm{SO}(p)(\mathrm{SO}(8-p))$ subalgebra while P_γ does onto the remainder $\mathrm{SO}(8)/(\mathrm{SO}(p)\times\mathrm{SO}(8-p))$. Here $\alpha=-1,\beta=p/(8-p)$ and $\gamma=(\alpha+\beta)/2$. The projectors of $\mathrm{SO}(p)\times\mathrm{SO}(8-p)$-invariant sectors are given in [1, appendix F of Part I] and corresponding A_1 and A_2 tensors are written as

$$A_1^{\;ij}=-\frac{4}{21}T_m^{\;ijm},\qquad A_{2l}^{\;ijk}=-\frac{4}{3}T_l^{\;[ijk]}. \tag{2.2}$$

We describe the potentials of various sectors of $\mathrm{SO}(p,8-p)$ and $\mathrm{CSO}(p,8-p)$ gaugings and are looking for any critical points in the latter. In previous paper [1], we considered gauged $\mathrm{SO}(p,8-p)$ supergravities with $\mathrm{SO}(p,8-p)$ gauge symmetry breaking it down to a solution with symmetry that is some subgroup of $\mathrm{SO}(p,8-p)$. That is, $\mathrm{SO}(p)\times\mathrm{SO}(8-p)$ for $\mathrm{SO}(p,8-p)$ gauging. In this section, we will take the subgroup to be G_2 for the $\mathrm{SO}(7,1)$

gauging, SU(3) for the SO(6, 2) gauging, SO(5) for the SO(5, 3) gauging and SO(3)$\times$SO(3) for SO(4, 4) gauging. All these subgroups are compact subgroup of noncompact SO(p, 8$-p$). Of course, the scalar potentials were obtained already in [3] and we will take different approach and see their equivalence. The 28-beins for given sectors of gauged supergravity theory in (2.1) are described completely in terms of some fields [7](See the appendix). The projectors of SO(p) $\times$ SO($8-p$) sectors are given in [7, appendix F]. Together with $\xi = -1$, the 28-beins for given sectors and the projectors for SO($p, 8-p$) gauged supergravity theories, one obtains the modified T-tensor (2.1). Finally one gets a scalar potential and a superpotential.

2.1 G_2 sector of SO(7, 1) gauging

It is known [2, 8, 3] that G_2-singlet space with a breaking of the SO(7) gauge subgroup of noncompact SO(7, 1) into a group which contains G_2 may be written as two real parameters λ and α. The vacuum expectation value of 56-bein $\mathcal{V}(x)$ for the G_2-singlet space that is invariant subspace under a particular G_2 subgroup of SO(7) can be parametrized by

$$\phi_{ijkl} = \lambda \cos\alpha \left(Y^{1\,+}_{ijkl} + Y^{2\,+}_{ijkl} \right) + \lambda \sin\alpha \left(Y^{1\,-}_{ijkl} + Y^{2\,-}_{ijkl} \right).$$

Here the completely anti-symmetric self-dual and anti-self-dual tensors which are invariant under SO(7)$^+$ and SO(7)$^-$ respectively are given in terms of $(Y^{1\,+}_{ijkl} + Y^{2\,+}_{ijkl})$ for the former and $(Y^{1\,-}_{ijkl} + Y^{2\,-}_{ijkl})$ for the latter[1] where their explicit forms are:

$$\begin{aligned} Y^{1\,\pm}_{ijkl} &= \varepsilon_{\pm} \left[\, (\delta^{1234}_{ijkl} \pm \delta^{5678}_{ijkl}) + (\delta^{1256}_{ijkl} \pm \delta^{3478}_{ijkl}) + (\delta^{3456}_{ijkl} \pm \delta^{1278}_{ijkl}) \, \right], \\ Y^{2\,\pm}_{ijkl} &= \varepsilon_{\pm} \left[-(\delta^{1357}_{ijkl} \pm \delta^{2468}_{ijkl}) + (\delta^{2457}_{ijkl} \pm \delta^{1368}_{ijkl}) + (\delta^{2367}_{ijkl} \pm \delta^{1458}_{ijkl}) + (\delta^{1467}_{ijkl} \pm \delta^{2358}_{ijkl}) \right], \end{aligned} \tag{2.3}$$

where $\varepsilon_+ = 1$ and $\varepsilon_- = i$ and $+$ gives the scalars and $-$ the pseudo-scalars of $\mathcal{N} = 8$ supergravity. The two scalars λ and α fields in the G_2-invariant flow parametrize a G_2-invariant subspace of the complete scalar manifold $E_{7(7)}/\,\mathrm{SU}(8)$ in the $d = 4, \mathcal{N} = 8$ supergravity. The 56-bein $\mathcal{V}(x)$ preserving G_2-invariance is a 56×56 matrix whose elements are some functions of two fields λ and α by exponentiating the above vacuum expectation value ϕ_{ijkl} of G_2-singlet space. Then 28-beins, u and v can be obtained and are 28×28 matrices given in the [7, appendix A] together with $\lambda' = \lambda$ and $\phi = \alpha$.

By applying all the data on u and v and the explicit form of the projectors P^{IJKL}_{σ} of SO(7)-invariant sector given in the [1, appendix F] to the equation (2.1), it turns out that $A_1^{\ ij}$ tensor for G_2 sector of this SO(7, 1) gauging with the condition $\xi = -1$ has two distinct complex eigenvalues, $z_1(\lambda, \alpha)$ and $z_2(\lambda, \alpha)$ with degeneracies 7, 1 respectively and has the following form

$$A_1^{\ ij} = \mathrm{diag}\,(z_1, z_1, z_1, z_1, z_1, z_1, z_1, z_2)\,,$$

[1]Sometimes these tensors are denoted by C^+_{ijkl} and C^-_{ijkl} respectively [8]. Note that G_2 is the common subgroup of SO(7)$^+$ and SO(7)$^-$. When $\alpha = 0$, it leads to the SO(7)$^+$-singlet space while $\alpha = \pi/2$ provides SO(7)$^-$-singlet space.

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where the eigenvalues $z_1(\lambda,\alpha)$ and $z_2(\lambda,\alpha)$ are functions of λ and α as follows

$$\begin{aligned}
z_1 &= \frac{1}{4}e^{-3i\alpha}\left(e^{i\alpha}p+q\right)\Big[3p^4q^2+3e^{6i\alpha}p^2q^4-2e^{i\alpha}p^3q\left(3p^2+2q^2\right)-\\
&\qquad -4e^{3i\alpha}pq\left(p^4-3p^2q^2+q^4\right)+e^{2i\alpha}p^2\left(3p^4-8p^2q^2-6q^4\right)-\\
&\qquad -2e^{5i\alpha}pq^3\left(2p^2+3q^2\right)+e^{4i\alpha}\left(-6p^4q^2-8p^2q^4+3q^6\right)\Big]\,,\\
z_2 &= \frac{1}{4}\Big(3p^7-7e^{-i\alpha}p^6q-21e^{-2i\alpha}p^5q^2-7e^{-3i\alpha}p^4q^3-7e^{-4i\alpha}p^3q^4-\\
&\quad -21e^{-5i\alpha}p^2q^5-7e^{-6i\alpha}pq^6+3e^{-7i\alpha}q^7\Big)
\end{aligned}\tag{2.4}$$

and we denote some hyperbolic functions of λ by the following quantities which will be used all the times in this paper

$$p\equiv\cosh\left(\frac{\lambda}{2\sqrt{2}}\right),\qquad q\equiv\sinh\left(\frac{\lambda}{2\sqrt{2}}\right).\tag{2.5}$$

The behavior of these eigenvalues of A_1 tensor looks similar to the G_2 sector of compact SO(8) gauging [7]. For G_2 sector of the non-compact SO(7,1) gauging, the expressions are more complicated. In particular, the magnitude of the eigenvalue z_2 plays the role of a superpotential of a scalar potential which will be discussed in section 4. The scalar potential can be obtained, by putting together all the components of A_1 tensor and A_2 tensor written explicitly in (A.1) and (A.2) and taking into account the multiplicities, as

$$\begin{aligned}
V(\lambda,\alpha) &= -g^2\left(\frac{3}{4}|A_1^{\;ij}|^2-\frac{1}{24}|A_{2i}^{\;\;jkl}|^2\right)\\
&= -g^2\left[\frac{3}{4}\times\left(7|z_1|^2+|z_2|^2\right)-\frac{1}{24}\times 6\left(7|y_{1,-}|^2+21|y_{2,-}|^2+28|y_{3,-}|^2\right)\right]\\
&= \frac{1}{2}g^2\left(c+vs\right)^2\left[\left(c+vs\right)\left(3c^2-8cvs+3v^2s^2\right)^2-14\left(c-vs\right)\left(c^2-4cvs+v^2s^2\right)\right]
\end{aligned}$$

that is exactly the same expression obtained by [3] [2] and we introduce the following quantities for simplicity

$$c\equiv\cosh\left(\frac{\lambda}{\sqrt{2}}\right),\qquad s\equiv\sinh\left(\frac{\lambda}{\sqrt{2}}\right),\qquad v\equiv\cos\alpha\,.\tag{2.6}$$

The analysis in [3] of the G_2-invariant critical points of the SO(7,1) gauging implies that there is no critical point while the G_2-invariant compact SO(8) potential possesses four critical points [2]: SO(8), $SO(7)^+$, $SO(7)^-$ and G_2.

[2] The G_2 sector of SO(7,1) scalar potential can be obtained also by analytic continuation from those sector of SO(8) scalar potential [3]. By replacing 56-bein $\mathcal{V}$ with $\mathcal{V}E(t)^{-1}$ and scaling by a factor of $e^{2\alpha t}$, the potential we are interested in is given by $e^{2\alpha t}V(\mathcal{V}E(t)^{-1})$ at $t=i\pi/(1+p/(8-p))$. Here $E(t)$ is the $\mathrm{SL}(8,\mathbb{R})$ element and the explicit relation between ξ and t is $\xi=e^{-(1+\frac{p}{8-p})t}$. By substituting the transformations $c\to\frac{1}{\sqrt{2}}(c-isv)$, $sv\to -i\frac{1}{\sqrt{2}}(c+isv)$ with $\alpha=-1$ and $t=i\pi/8(p=7)$ into the G_2 sector of SO(8) scalar potential [2] $V=2g^2\left[(7v^4-7v^2+3)c^3s^4+(4v^2-7)v^5s^7+c^5s^2+7v^3c^2s^5-3c^3\right]$ and multiplying the factor $e^{2\alpha t}=e^{-i\pi/4}$, we get the above G_2 sector of SO(7,1) scalar potential.

2.2 SU(3) sector of SO(6,2) gauging

Similarly the parametrization for the SU(3)-singlet space [2, 3] that has an invariant subspace under a particular SU(3) subgroup of SO(6)(= SU(4)) gauge subgroup of noncompact SO(6,2) can be described by

$$\phi_{ijkl} = \lambda \cos\alpha Y^{1\,+}_{ijkl} + \lambda \sin\alpha Y^{1\,-}_{ijkl} + \lambda' \cos\phi Y^{2\,+}_{ijkl} + \lambda' \sin\phi Y^{2\,-}_{ijkl}\,,$$

where the scalar and pseudo-scalar singlets of SU(3) are given in (2.3) as before. When we put the constraint of $\lambda' = \lambda$ and $\phi = \alpha$, then we get previous G_2-invariant sector. The four scalars $\lambda, \lambda', \alpha$ and ϕ fields in the SU(3)-invariant flow parametrize a SU(3)-invariant subspace of the complete scalar manifold. Then the 56-bein $\mathcal{V}(x)$ for SU(3)-invariance is a function of $\lambda, \lambda', \alpha$ and ϕ and 28-beins u, v are also some functions of these four fields: we refer to the [7, appendix A] for explicit relations. Now we substitute all the expressions of u and v and the projectors P_σ^{IJKL} of SO(6) $\times$ SO(2)-invariant sector given in the of [1, appendix F] to the defining equation (2.1). Then one obtains that ${A_1}^{ij}$ tensor for SU(3) sector of this SO(6,2) gauging with $\xi = -1$ has three different complex eigenvalues $z_1(\lambda, \lambda', \alpha, \phi), z_2(\lambda, \lambda', \alpha, \phi)$ and $z_3(\lambda, \lambda', \alpha, \phi)$ with multiplicities 6, 1, 1 respectively as follows

$${A_1}^{ij} = \mathrm{diag}\,(z_1, z_1, z_1, z_1, z_1, z_1, z_2, z_3)\,,$$

where their explicit dependence on those parameters are more involved when we compare with the one of the SU(3) sector [7] of compact SO(8) gauging but their structure looks similar to those in compact case and are given

$$\begin{aligned}
z_1 = {} & \frac{1}{2} e^{-i(\alpha+2\phi)} \Big[p^2 q r^2 t^2 + e^{4i\phi} p^2 q r^2 t^2 + e^{3i\alpha} p q^2 r^2 t^2 + e^{3i\alpha+4i\phi} p q^2 r^2 t^2 - \\
& - e^{2i\alpha} q \left(2p^2 + q^2\right) r^2 t^2 - e^{2i(\alpha+2\phi)} q \left(2p^2 + q^2\right) r^2 t^2 - \\
& - e^{i\alpha} p \left(p^2 + 2q^2\right) r^2 t^2 - e^{i(\alpha+4\phi)} p \left(p^2 + 2q^2\right) r^2 t^2 - \\
& - e^{2i\phi} p^2 q \left(r^4 + 4r^2 t^2 + t^4\right) - e^{3i\alpha+2i\phi} p q^2 \left(r^4 + 4r^2 t^2 + t^4\right) + \\
& + e^{i(\alpha+2\phi)} p \left(-2q^2 \left(r^4 + t^4\right) + p^2 \left(r^4 - 4r^2 t^2 + t^4\right)\right) + \\
& + e^{2i(\alpha+\phi)} \left(-2p^2 q \left(r^4 + t^4\right) + q^3 \left(r^4 - 4r^2 t^2 + t^4\right)\right) \Big], \\
z_2 = {} & \frac{1}{2} e^{-3i\alpha} \left(e^{i\alpha} p + q\right) \Big(e^{2i\alpha} p^2 r^4 - 4 e^{i\alpha} p q r^4 + q^2 r^4 - 6 e^{2i(\alpha+\phi)} p^2 r^2 t^2 - \\
& - 6 e^{2i\phi} q^2 r^2 t^2 + e^{2i(\alpha+2\phi)} p^2 t^4 - 4 e^{i(\alpha+4\phi)} p q t^4 + e^{4i\phi} q^2 t^4 \Big), \\
z_3 = {} & \frac{1}{2} e^{-i(3\alpha+4\phi)} \left(e^{i\alpha} p + q\right) \Big(e^{2i(\alpha+2\phi)} p^2 r^4 - 4 e^{i(\alpha+4\phi)} p q r^4 + e^{4i\phi} q^2 r^4 - 6 e^{2i(\alpha+\phi)} p^2 r^2 t^2 - \\
& - 6 e^{2i\phi} q^2 r^2 t^2 + e^{2i\alpha} p^2 t^4 - 4 e^{i\alpha} p q t^4 + q^2 t^4 \Big)
\end{aligned} \tag{2.7}$$

together with the following quantities and (2.5)

$$r \equiv \cosh\left(\frac{\lambda'}{2\sqrt{2}}\right), \qquad t \equiv \sinh\left(\frac{\lambda'}{2\sqrt{2}}\right). \tag{2.8}$$

Although the structures of these eigenvalues are more involved, their degeneracies resemble the SU(3) sector of compact SO(8) gauging. In this case also, the magnitude of complex z_3

will give rise to a superpotential of a scalar potential which will be discussed later. Then the effective nontrivial scalar potential, by plugging the A_1 tensor and A_2 tensor given in (A.4) and (A.5) into the definition of potential and counting the degeneracies correctly, becomes

$$\begin{aligned}
V &= -g^2\left(\frac{3}{4}|A_1{}^{ij}|^2 - \frac{1}{24}|A_{2i}{}^{jkl}|^2\right) \\
&= -g^2\Bigg[\frac{3}{4}\times\left(6|z_1|^2+|z_2|^2+|z_3|^2\right) - \\
&\qquad -\frac{1}{24}\times 6\Big(3|y_{1,-}|^2+3|y_{2,-}|^2+4|y_{3,-}|^2+12|y_{4,-}|^2+12|y_{5,-}|^2+4|y_{6,-}|^2+ \\
&\qquad\qquad +6|y_{7,-}|^2+12|y_{8,-}|^2\Big)\Bigg] \\
&= \frac{1}{2}g^2\Big\{s'^4\left[(c+vs)\left(2xc-(x-3)\,vs\right)^2-3\,(x-1)\left((x+1)\,c+2vs\right)\right]+ \\
&\qquad +s'^2\Big[2\,(c+vs)\left(2c^2+2\,(3x-1)\,cvs-(3x-5)\,v^2s^2\right)+ \\
&\qquad\qquad +6\left((x+1)\,c-(x-3)\,vs\right)\Big]+12vs\Big\}
\end{aligned}$$

which is the same result of [3][3] and we introduce the following quantities as well as the relations (2.6)

$$c' \equiv \cosh\left(\frac{\lambda'}{\sqrt{2}}\right), \qquad s' \equiv \sinh\left(\frac{\lambda'}{\sqrt{2}}\right), \qquad x \equiv \cos 2\phi. \tag{2.9}$$

It was known [3] that there is no SU(3)-invariant critical point in SU(3) sector of SO(6, 2) gauging. Although the compact SO(8) potential has six SU(3)-invariant critical points [2], the SO(6, 2) potential has none. In other words, there are two additional critical points, $SU(4)^-(= SO(6)^-)$ and SU(3) × U(1) critical points, besides the four G_2-invariant critical points we have mentioned in the subsection 2.1.

2.3 SO(5) sector of SO(5, 3) gauging

One can construct SO(5)-singlets [9, 3] parametrized by

$$\phi_{ijkl} = \lambda\left(X_1^+ + X_2^+ + X_3^+\right) + \mu\left(X_1^+ + X_4^+ + X_5^+\right) + \rho\left(X_1^+ - X_6^+ - X_7^+\right), \tag{2.10}$$

where λ, μ and ρ characteristic of SO(5)-singlets are three real parameters and self-dual four-forms are

$$\begin{aligned}
X_1^+ &= \frac{1}{2}\left(\delta^{1234}_{ijkl}+\delta^{5678}_{ijkl}\right), & X_2^+ &= \frac{1}{2}\left(\delta^{1256}_{ijkl}+\delta^{3478}_{ijkl}\right), & X_3^+ &= \frac{1}{2}\left(\delta^{1278}_{ijkl}+\delta^{3456}_{ijkl}\right), \\
X_4^+ &= -\frac{1}{2}\left(\delta^{1357}_{ijkl}+\delta^{2468}_{ijkl}\right), & X_5^+ &= \frac{1}{2}\left(\delta^{1368}_{ijkl}+\delta^{2457}_{ijkl}\right), & X_6^+ &= \frac{1}{2}\left(\delta^{1458}_{ijkl}+\delta^{2367}_{ijkl}\right), \\
X_7^+ &= \frac{1}{2}\left(\delta^{1467}_{ijkl}+\delta^{2358}_{ijkl}\right).
\end{aligned} \tag{2.11}$$

[3] By plugging the transformations of λ and α: $c \to -ivs,\ sv \to -ic$ while λ' and ϕ remain unchanged with $\alpha = -1$ and $t = i\pi/4 (p = 6)$ into the SU(3) sector of SO(8) scalar potential given in [2] and multiplying the factor $e^{-i\pi/2}$, the SU(3) sector of SO(6, 2) scalar potential can be obtained by analytic continuation from those sector of SO(8) scalar potential [3].

In this case, the SO(5) singlet space breaks the SO(5) gauge subgroup of noncompact SO(5,3) into a group which contains SO(5). The three scalars λ, μ and ρ fields in the SO(5)-invariant flow parametrize a SO(5)-invariant subspace of the complete scalar manifold $E_{7(7)}/\,\mathrm{SU}(8)$ in $d = 4, \mathcal{N} = 8$ supergravity. The 56-bein preserving SO(5)-invariance and 28-beins are functions of three fields λ, μ and ρ and their explicit form is given in the [7, appendix B]. The eigenvalues of A_1 tensor are classified by a single real one, $z_1(\lambda, \mu, \rho)$ which plays the role of a superpotential(which will be studied later) after we are plugging the expressions of u and v and the projectors P_σ^{IJKL} of $\mathrm{SO}(5) \times \mathrm{SO}(3)$-invariant sector given in the [1, appendix F] to the equation (2.1) with $\xi = -1$:

$$A_1{}^{ij} = \mathrm{diag}\,(z_1, z_1, z_1, z_1, z_1, z_1, z_1, z_1)\,,$$

where we write them in terms of new variables as in the case of SO(5) sector of compact SO(8) gauging [9]

$$z_1(\lambda, \mu, \rho) = \frac{1}{8\sqrt{uvw}} \left(5 - u^2 v^2 + \text{two cyclic permutations}\right), \tag{2.12}$$

where we define

$$u \equiv e^{\lambda/\sqrt{2}}, \qquad v \equiv e^{\mu/\sqrt{2}}, \qquad w \equiv e^{\rho/\sqrt{2}}\,. \tag{2.13}$$

When we compare with SO(5) sector of SO(8) scalar potential, there exists a relative sign change in the above. Finally we will arrive at the scalar potential for SO(5)-singlets by substituting all the components of A_1 tensor and A_2 tensor given in (A.7) and (A.8) and taking the multiplicities appropriately:

$$\begin{aligned}
V(\lambda, \mu, \rho) &= -g^2 \left(\frac{3}{4}|A_1{}^{ij}|^2 - \frac{1}{24}|A_{2i}{}^{jkl}|^2\right) \\
&= -g^2 \left[\frac{3}{4} \times 8|z_1|^2 - \frac{1}{24} \times 6\left(16|y_{1,-}|^2 + 16|y_{2,-}|^2 + 16|y_{3,-}|^2 + 8|y_{4,-}|^2\right)\right] \\
&= \frac{1}{8} g^2 \left(\frac{u^3 v^3}{w} + \frac{10uv}{w} - 2uvw^3 + \text{two cyclic permutations} - \frac{15}{uvw}\right)
\end{aligned}$$

that was observed also in [3] [4] and note that the difference from SO(8) potential restricted to SO(5) scalar singlets is the change of sign in the coefficient of uv/w in the above potential. It was found that there exists one critical point of this scalar potential when $\lambda = \mu = \rho$(Note that the SO(5)-singlet structure (2.10) should preserve SO(5,3)-invariance characterized by self-dual antisymmetric four-form tensor $X_{5,3}^{+IJKL}$ written in the [1, appendix A] and the condition $\lambda = \mu = \rho$ should be satisfied in order to require that (2.10) be proportional to $X_{5,3}^{+IJKL}$) and the cosmological constant becomes $V = 2 \times 3^{1/4} g^2$ with $u = 3^{-1/4}$. In this subspace the above potential reduces to SO(5,3) scalar potential $V_{5,3}$ with $\xi = -1$

[4]In this case, we do not need to use Baker-Hausdorff formula because the SO(3) action in the 56-beins $\mathcal{V}$ commutes with $E(t)^{-1}$ for SO(5,3). Therefore the SO(5,3) potential is independent of the action of SO(3). The Lie algebra element generating $E(t)$ can be obtained by setting $\lambda = \mu = \rho$. By substituting the transformations $\frac{\lambda}{\sqrt{2}} \to \frac{\lambda}{\sqrt{2}} - \frac{1}{4} i\pi$, $\frac{\mu}{\sqrt{2}} \to \frac{\mu}{\sqrt{2}} - \frac{1}{4} i\pi$, $\frac{\rho}{\sqrt{2}} \to \frac{\rho}{\sqrt{2}} - \frac{1}{4} i\pi$ and multiplying the factor $e^{-3\pi i/4}$ into the SO(5) sector of SO(8) scalar potential [9] one can get this SO(5) sector of SO(5,3) scalar potential [3].

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in [1] with the identification of $s = -\frac{3}{2\sqrt{2}}\lambda$ where s is a scalar field defined in [1]. Note that the SO(5) sector of compact SO(8) gauging has two critical points [9]: a trivial maximally supersymmetric SO(8) critical point and a nonsupersymmetric SO(7)-invariant critical point. All of these are AdS critical points.

2.4 SO(3) $\times$ SO(3) sector of SO(4,4) gauging

It is known that SO(3) $\times$ SO(3)-singlet space with a breaking of the SO(4) $\times$ SO(4) into SO(3) $\times$ SO(3) maybe written as

$$\phi_{ijkl} = S(\lambda^\alpha X_\alpha^+), \qquad \alpha = 1, 2, \ldots, 7\,.$$

Here the action S is SO(3) $\times$ SO(3) subgroup of SU(8) on its 70-dimensional representation in the space of self-dual four-forms and is given in [10]. Self-dual four forms X_α^+ are given in (2.11). The λ^α's that are seven real parameters parametrize SO(3) $\times$ SO(3)-invariant subspace of full scalar manifold in $d = 4, \mathcal{N} = 8$ supergravity. The 56-bein $\mathcal{V}$ and 28-beins u, v are some functions of these parameters and they appear in [7]. After we are plugging the expressions of u and v and the projectors P_σ^{IJKL} of SO(4) $\times$ SO(4)-invariant sector given in the [1, appendix F] to the equation (2.1) with $\xi = -1$, then one obtains A_1 tensor classified by eight distinct complex ones $z_i (i = 1, 2, \ldots, 8)$. Of course, the structure of these expressions is complicated and the scalar potential can be obtained as usual. However, it is not very much illuminating to present here. It was checked in [3] that in this case also there is no critical point.

3. The potentials of CSO$(p, 8-p)$ gauged supergravity

In this section, we will take the subgroup to be G_2 for the CSO(7,1) gauging, SU(3) for the CSO(6,2) gauging, SO(5) for the CSO(5,3) gauging. The 28-beins for given sectors of gauged supergravity theory in (2.1) are described completely in terms of some fields [7]. The projectors of SO(p) $\times$ SO$(8-p)(p = 7, 6, 5)$ sectors are given in the [7, appendix F]. With $\xi = 0$, 28-beins for given sectors and projectors for CSO$(p, 8-p)$ gauged supergravity theory, one obtains the modified T-tensor (2.1). Finally one gets a new scalar potential by using the definition of scalar potential given by A_1 and A_2 tensors. In particular, the SU(3) sector of CSO(6,2) gauging provides three AdS critical points which are our new findings.

3.1 G_2 sector of CSO(7,1) gauging

By applying all the data on u, v which are the same as those in previous SO(7,1) gauging and the projectors P_σ^{IJKL} of SO(7)-invariant sector given in the [1, appendix F] to (2.1), A_1 tensor for G_2 sector of this CSO(7,1) gauging with the condition $\xi = 0$ has two distinct complex eigenvalues, $z_1(\lambda, \alpha)$ and $z_2(\lambda, \alpha)$ with degeneracies 7, 1 respectively. In this case, G_2-singlet space breaks the SO(7) gauge group of non-semi-simple CSO(7,1) into a group that contains G_2. We emphasize that the only difference between G_2 sectors of previous SO(7,1) gauging and present CSO(7,1) gauging is that the parameter ξ is -1 for the former and 0 for the latter. Otherwise 28-beins and projectors are the same. Then the A_1 tensor has the following form

$$A_1{}^{ij} = \mathrm{diag}\,(z_1, z_1, z_1, z_1, z_1, z_1, z_1, z_2)\,,$$

where the two distinct eigenvalues $z_1(\lambda,\alpha)$ and $z_2(\lambda,\alpha)$ are given by

$$\begin{aligned} z_1 &= \frac{1}{8} e^{-3i\alpha}\left(e^{i\alpha}p+q\right)\left(p-e^{i\alpha}q\right)^2 \times \\ &\quad \times \left[7p^2q^2 + 7e^{4i\alpha}p^2q^2 - 10e^{i\alpha}pq + 10e^{3i\alpha}pq + 7e^{2i\alpha}\left(p^4 - 4p^2q^2 + q^4\right)\right], \\ z_2 &= \frac{7}{8} e^{-7i\alpha}\left(-e^{i\alpha}p+q\right)^4\left(e^{i\alpha}p+q\right)^3, \end{aligned} \tag{3.1}$$

where p and q are defined as (2.5). The behavior of the eigenvalues of A_1 tensor shares with those sectors in SO(8) and SO(7,1) gaugings. The superpotential for this theory can be read off from the expression of z_2. Now it is straightforward to find out the scalar potential from A_1 tensor and A_2 tensor written in (A.1) and (A.3) like we did before

$$\begin{aligned} V(\lambda,\alpha) &= -g^2\left[\frac{3}{4}\times\left(7|z_1|^2+|z_2|^2\right) - \frac{1}{24}\times 6\left(7|y_{1,0}|^2 + 21|y_{2,0}|^2 + 28|y_{3,0}|^2\right)\right] \\ &= \frac{7}{8}g^2\left(-12+7c^2-7v^2s^2\right)(c-vs)^3(c+vs)^2\,, \end{aligned} \tag{3.2}$$

where c, s and v are defined as in (2.6). One can obtain also the G_2 sector of the CSO(7,1) theory by analytic continuation as follows: As done in obtaining G_2 sector of SO(7,1) potential from those sector of SO(8) scalar potential, by substituting the transformations [3]

$$c \to (c\cosh 2t - sv\sinh 2t)\,, \qquad sv \to (-c\sinh 2t + sv\cosh 2t)$$

with $\alpha = -1$ into the G_2 sector of SO(8) scalar potential [2], multiplying the factor $e^{2\alpha t}$ and taking $t \to \infty$, we get the above G_2 sector of CSO(7,1) scalar potential. Note that $\xi = e^{-(1+\frac{p}{8-p})t}$. Now we are looking for any critical points of this scalar potential if there are. Differentiating (3.2) with respect to field α, one obtains

$$[s(c-sv)^2(c+sv)(12c-7c^3-(-60+49c^2)sv+7cs^2v^2+49s^3v^3)]\sin\alpha = 0\,.$$

There exist two possibilities either $\sin\alpha = 0$ or the expression in the brackets vanishes. Let us consider the first case.

$\sin\alpha = 0$. In terms of v, this implies that $v = 1$ or $v = -1$. Since v appears the combination of vs in a scalar potential V (3.2), the case of $v = -1$ maybe obtained from $v = 1$ by letting $\lambda \to -\lambda$. So we need to analyze the case of $v = 1$ only. In this subspace, the scalar potential (3.2) reduces to

$$V = -\frac{35}{8}g^2(c-s) = -\frac{35}{8}g^2 e^{-\lambda/\sqrt{2}}$$

which does not have any critical points. Let us describe the second case.

$\sin\alpha \neq 0$. Let us change the independent variables in the scalar potential V (3.2) as follows:

$$A = c\,, \qquad B = vs\,,$$

where it is easy to see that this transformation is nonsingular due to $\sin\alpha \neq 0$. One can find there are no solutions satisfying $\partial_A V = \partial_B V = 0$ where we used the fact that $|A| > |B|$.

3.2 SU(3) sector of CSO(6,2) gauging

In this case, SU(3)-singlet space breaks the SO(6) gauge group of non-semi-simple CSO(6,2) into a group that contains SU(3). With all the data on u, v and the projectors P_σ^{IJKL} of $SO(6) \times SO(2)$-invariant sector given in the [1, appendix F] that are same as those in SO(6,2) gauging, ${A_1}^{ij}$ tensor for SU(3) sector of this CSO(6,2) gauging with the condition $\xi = 0$ has three distinct complex eigenvalues with degeneracies 6, 1, 1 respectively and has the following form

$${A_1}^{ij} = \mathrm{diag}\,(z_1, z_1, z_1, z_1, z_1, z_1, z_2, z_3)\,,$$

where they are given in terms of four papramaters

$$\begin{aligned}
z_1 &= \frac{1}{4} e^{-i(\alpha+2\phi)} \left(p - e^{i\alpha} q\right) \times \\
&\quad \times \Big[3pqr^2t^2 - 3e^{2i\alpha} pqr^2t^2 + 3e^{4i\phi} pqr^2t^2 - 3e^{2i(\alpha+2\phi)} pqr^2t^2 - e^{i\alpha} r^2 t^2 - e^{i(\alpha+4\phi)} r^2 t^2 - \\
&\qquad - e^{2i\phi} pq \left(r^4 + 4r^2t^2 + t^4\right) + e^{2i(\alpha+\phi)} pq \left(r^4 + 4r^2t^2 + t^4\right) + \\
&\qquad + e^{i(\alpha+2\phi)} \left(3r^4 - 4r^2t^2 + 3t^4\right) \Big]\,, \\
z_2 &= \frac{3}{4} e^{-3i\alpha} \left(-e^{i\alpha} p + q\right)^2 \left(e^{i\alpha} p + q\right) \left(r^2 - e^{2i\phi} t^2\right)^2\,, \\
z_3 &= \frac{3}{4} e^{-i(3\alpha+4\phi)} \left(-e^{i\alpha} p + q\right)^2 \left(e^{i\alpha} p + q\right) \left(-e^{2i\phi} r^2 + t^2\right)^2
\end{aligned} \tag{3.3}$$

with (2.5) and (2.8). The scalar potential from the A_1 tensor and A_2 tensor given in (A.4) and (A.6) leads to

$$\begin{aligned}
V &= -g^2 \Big[\frac{3}{4} \times \left(6|z_1|^2 + |z_2|^2 + |z_3|^2\right) - \\
&\qquad - \frac{1}{24} \times 6 \Big(3|y_{1,0}|^2 + 3|y_{2,0}|^2 + 4|y_{3,0}|^2 + 12|y_{4,0}|^2 + 12|y_{5,0}|^2 + 4|y_{6,0}|^2 + \\
&\qquad\qquad + 6|y_{7,0}|^2 + 12|y_{8,0}|^2 \Big) \Big] \\
&= \frac{3}{8} g^2\, (c - s\,v) \left[-2 + s'^2\,(x-1)\right] \left[4 + s'^2 \left(-2 + 3c^2 - 3s^2\,v^2\right)(x-1)\right]
\end{aligned} \tag{3.4}$$

together with (2.6) and (2.9). By plugging the transformations of λ and α [3],

$$c \to (c\cosh 2t - sv\sinh 2t)\,, \qquad sv \to (-c\sinh 2t + sv\cosh 2t)$$

with $\alpha = -1$ into the SU(3) sector of SO(8) scalar potential [2], multiplying the factor e^{-2t} and taking the limit of $t \to \infty$, the SU(3) sector of CSO(6,2) scalar potential can be obtained also by analytic continuation from those sector of SO(8) scalar potential [3]. We describe the structure of critical points of this potential if they exist. Differentiating (3.4) with respect to field α, one obtains

$$
\begin{aligned}
&s\left[-2+s'^2(-1+x)\right]\times\\
&\times\left[4+6ss'^2(-1+x)\cos\alpha(c-s\cos\alpha)+s'^2(-1+x)(-2+3c^2-3s^2\cos^2\alpha)\right]\sin\alpha=0\,.
\end{aligned}
$$

There exists two possibilities either $\sin\alpha=0$ or the expression in the brackets vanishes. Let us describe the first case.

3.2.1 $\sin\alpha = 0$

In terms of v, this implies that $v=1$ or $v=-1$. Since v appears the combination of vs in a scalar potential V (3.4), the case of $v=-1$ maybe obtained from $v=1$ by letting $\lambda\to-\lambda$. So we need to analyze the case of $v=1$ only. In this subspace, the scalar potential reduces to

$$
V=\frac{3}{8}g^2\,(c-s)\left(-2+s'^2(-1+x)\right)\left[4+s'^2(-1+x)\right]\qquad\text{at}\qquad\alpha=0\,. \tag{3.5}
$$

Differentiating (3.5) with respect to field ϕ, one obtains

$$
\frac{\partial V}{\partial\phi}=\frac{3}{2}g^2\,(c-s)\,s'^2\left(-1+2s'^2\sin^2\phi\right)\sin2\phi=0\,.
$$

There exist four cases we have to consider. Let us describe the case of $\phi=0,\pi/2$ first.

$\sin 2\phi = 0$. In this subspace , the scalar potential will be

$$
\begin{aligned}
V&=-3g^2e^{-\lambda/\sqrt{2}}\,,\qquad\text{at}\qquad\alpha=0\,,\quad\phi=0\,,\\
V&=\frac{3}{2}g^2\,(c-s)\left(1+s'^2\right)\left(-2+s'^2\right),\qquad\text{at}\qquad\alpha=0\,,\quad\phi=\frac{\pi}{2}\,.
\end{aligned}
$$

We do not have any critical points in the first potential and for the second case it is easy to see that the conditions of $\partial_\lambda V=\partial_{\lambda'}V=0$ will provide an imaginary solution for λ' and therefore there are no critical points.

$c = s$. There is no real solution for $c=s$ and therefore there is no critical point.

$s' = 0$. The scalar potential becomes

$$
V=-3g^2e^{-\lambda/\sqrt{2}}\qquad\text{at}\qquad\lambda'=0
$$

and there is no critical point.

$-1+2s'^2\sin^2\phi = 0$. One can substitute ϕ or λ' satisfying this condition into the potential (3.4) and we get by eliminating ϕ

$$
V=-\frac{27}{8}g^2e^{-\lambda/\sqrt{2}}\qquad\text{at}\qquad\alpha=0\,.
$$

We do not have any critical points. Now we move on the second case.

3.2.2 $\sin\alpha \neq 0$

Now let us consider the second case of $\sin\alpha \neq 0$. Then due to the negativeness of $-2 + s'^2(-1+x)$, there are two cases we have to study. We will describe the first case.

$s = 0$. Let us plug $\lambda = 0$ into the scalar potential (3.4) and then we get

$$V(\lambda', \phi) = \frac{3}{8} g^2 \left(-2 + s'^2(-1+x)\right) \left(4 + s'^2(-1+x)\right) \qquad \text{at} \qquad \lambda = 0 \,. \tag{3.6}$$

One can easily get the solutions by differentiating V (3.6) with respect to λ' and ϕ and putting zero respectively:

$$\text{i) } \lambda' = \pm\frac{1}{\sqrt{2}} \log\left(2+\sqrt{3}\right), \qquad \phi = \pm\frac{\pi}{2} \qquad \text{ii) } \lambda' = 0\,, \qquad \text{iii) } \phi = 0\,.$$

One can get an extra condition of $\alpha = \pi/2(\text{or } 3\pi/2)$ when we perform a differentiation of the scalar potential with respect to λ and evaluate it at the above critical values. By requiring this should vanish, one gets $\alpha = \pi/2(\text{or } 3\pi/2)$. Now one can evaluate the potential at each critical point. Now we summarize them as follows:[5]

$$\begin{aligned}
V &= -\frac{27}{8} g^2\,, \text{ at } \lambda = 0\,, && \lambda' = \pm\frac{1}{\sqrt{2}} \log\left(2+\sqrt{3}\right), && \alpha = \frac{\pi}{2}\,, && \phi = \pm\frac{\pi}{2} \\
V &= -3g^2\,, \quad \text{at } \lambda = 0\,, && \lambda' = \text{arbitrary}, && \alpha = \frac{\pi}{2}\,, && \phi = 0\,, \\
V &= -3g^2\,, \quad \text{at } \lambda = 0\,, && \lambda' = 0\,, && \alpha = \frac{\pi}{2}\,, && \phi = \text{arbitrary}\,.
\end{aligned} \tag{3.7}$$

In the first critical point in the above, one finds that the A_1 tensor eigenvalues are $7/8$ with six degeneracies and $9/8$ with two degeneracies and neither eigenvalue satisfies $W = \sqrt{-V/6g^2}$ and so the supersymmetry is completely broken. In the last two critical points, the A_1 tensor eigenvalues are $3/4$ with eight degeneracies that does not satisfies $W = \sqrt{-V/6g^2}$ also. We draw the scalar potential $V(\lambda', \phi)$ given by (3.6) in figure 1 in order to visualize the structure of these critical points. The four critical points at which the cosmological constants becomes $-\frac{27}{8}g^2$ correspond to a local minimum while a critical point at which $\phi = 0$ has flat direction in λ' direction and a critical point at which $\lambda' = 0$ has flat direction in ϕ direction.

At $\lambda = 0$ and $\phi = \frac{\pi}{2}$, the scalar potential further reduces to and is described in figure 1

$$\begin{aligned}
V(\lambda') &= \frac{3}{4} g^2 \cosh^2\left(\frac{\lambda'}{\sqrt{2}}\right)\left(-5 + \cosh\left(\sqrt{2}\lambda'\right)\right) \\
&= \frac{3}{8} g^2 \left(p^2 - 4p - 5\right), \qquad p \equiv \cosh(4\Lambda), \qquad \lambda' = 2\sqrt{2}\Lambda
\end{aligned} \tag{3.8}$$

which is proportional to the scalar potential of SU(3) sector of SO(6) gauging in five dimensions [11]. In the context of five dimensional viewpoint, this potential has two AdS critical points. One is a maximally supersymmetric critical point at $\Lambda = 0$ corresponding

[5] Before analyzing these analytic solutions, there was an attempt to get some of these by a numerical method. We thank T. Fischbacher to help us.

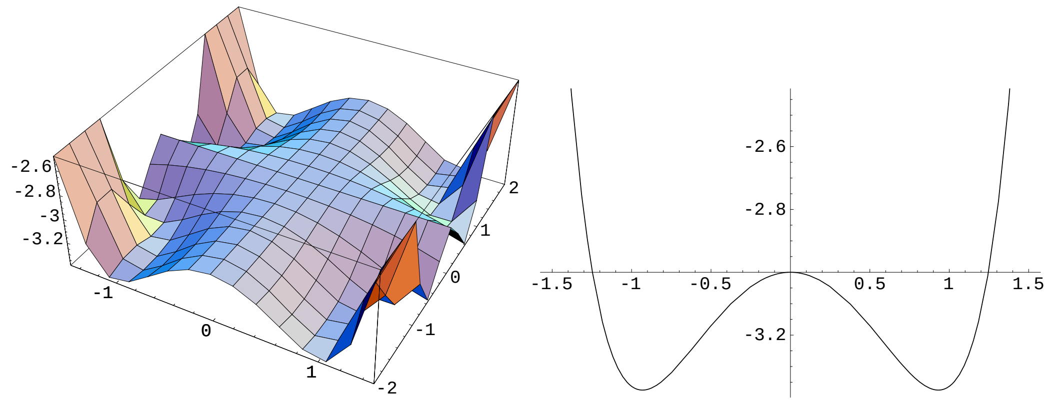

Figure 1: The plots of the scalar potential $V(\lambda', \phi)$(left) at $\lambda = 0, \alpha = \pi/2$ and the scalar potential $V(\lambda')$(right) at $\lambda = 0, \alpha = \pi/2, \phi = \pi/2$ in the 4-dimensional gauged supergravity. The axes (λ', ϕ) in the left are two vevs that parametrize the SU(3) invariant manifold in the 28-beins of the theory. The four critical points in (3.6) are located at $\lambda' = \pm\frac{1}{\sqrt{2}} \log\left(2+\sqrt{3}\right) \approx \pm 0.93123$ and $\phi = \pm\frac{\pi}{2} \approx \pm 1.5708$ whereas the other critical points where the potentials are flat in these directions are located at an arbitrary point on the line of $\phi = 0$ or at an arbitrary point on the line of $\lambda' = 0$. In the right scalar potential (3.8) we further restricted to the slice of $\phi = \pi/2$. It turns out that this potential coincides with the one in SU(3) invariant sector of SO(6) gauging in five dimensional supergravity. We have set the gauge coupling g in the scalar potential as $g = 1$.

to the above critical point at which the potential has $-3g^2$ in four dimensions and the other is a nonsupersymmetric critical point at $p = \cosh(4\Lambda) = 2$, corresponding to the critical point at which the potential is $-\frac{27}{8}g^2$ in four dimensions, breaking the SO(6) gauge symmetry into SU(3) $\times$ U(1). The relevant operators in the four dimensional $\mathcal{N} = 4$ super Yang-Mills are mapped to the scalars in the supergravity multiplet. The existence of an unstable nonsupersymmetric SU(3)-invariant background of $AdS_5 \times \mathbf{S}^5$ of type IIB string theory was described in [12, 13] from the mass spectrum of the low-lying states in this SU(3)-invariant supergravity solution.

Some time ago, a noncompact SO(6)$^* =$ SU(3,1) gauging in five dimensions was constructed [14] and the scalar potential has a critical point that breaks the gauge symmetry down to SU(3) $\times$ U(1). The potential is obtained by replacing p by $-p$ in (3.8) and has a critical point at $\Lambda = 0$ at which the potential vanishes. Recently, it was shown that dimensionally reducing the SO(6)* theory to four dimensional theory and dualizing the graviphoton gave the CSO(6,2)* gauging which is a non-semi-simple contractions of SO(8)$^* = SO(6,2)$ in analogy with the contraction CSO$(p, 8-p)$ of SO(8) [15].

Let us close this subsection by considering the second case.

$4+6ss'^2(-1+x)\cos\alpha(c-s\cos\alpha)+s'^2(-1+x)(-2+3c^2-3s^2\cos^2\alpha) = 0$. Let us change the independent variables in the scalar potential V (3.4) as follows:

$$A = c\,, \qquad B = vs\,.$$

Then one can compute the derivatives of V with respect to fields A, B, s' and x. By requiring that those are vanishing, one has no real solutions in this case where we used that fact that $|A| > |B|$.

3.3 SO(5) sector of CSO(5,3) gauging

It turns out that the eigenvalues of A_1 tensor are classified by a single real one, $z_1(\lambda, \mu, \rho)$

$$A_1{}^{ij} = \mathrm{diag}\,(z_1, z_1, z_1, z_1, z_1, z_1, z_1, z_1)\,, \qquad z_1 = \frac{5}{8\sqrt{uvw}} \tag{3.9}$$

together with (2.13). The scalar potential is given by with the data of A_2 tensor in (A.9) and (A.10)

$$V(\lambda, \mu, \rho) = -g^2 \left[\frac{3}{4} \times 8|z_1|^2 - \frac{1}{24} \times 6 \left(48|y_{1,0}|^2 + 8|y_{2,0}|^2\right) \right] = -\frac{15}{8uvw} g^2\,.$$

There is no critical point in this potential. The SO(3) action in the 56-beins $\mathcal{V}$ commutes with $E(t)^{-1}$ for CSO(5,3) and the CSO(5,3) potential is independent of the action of SO(3). The Lie algebra element generating $E(t)$ can be obtained by setting $\lambda = \mu = \rho$ similarly. By substituting the transformations

$$\frac{\lambda}{\sqrt{2}} \to \frac{\lambda}{\sqrt{2}} - \frac{2t}{3}\,, \qquad \frac{\mu}{\sqrt{2}} \to \frac{\mu}{\sqrt{2}} - \frac{2t}{3}\,, \qquad \frac{\rho}{\sqrt{2}} \to \frac{\rho}{\sqrt{2}} - \frac{2t}{3}\,,$$

multiplying the factor e^{-2t} and taking $t \to \infty$ into the SO(5) sector of SO(8) scalar potential [9] one can get also this SO(5) sector of CSO(5,3) scalar potential.

4. Domain wall in SO$(p, 8-p)$ and CSO$(p, 8-p)$ gaugings

One of the eigenvalues of A_1 tensor for given sectors of gauged supergravity theory allows us to write a superpotential for a scalar potential. In order to find domain-wall solutions for the theory we have considered so far, it is necessary to express the energy functional in terms of complete squares in the usual sense. Since now one can reorganize the scalar potential in terms of a sum of squares of superpotential and the derivatives of superpotential with respect to the fields, it leads to the minimization of energy functional and one gets domain wall solutions without any difficulty. This observation is exactly the same as the one in the compact SO(8) gauged supergravity theory [7].

4.1 G_2 sectors of SO(7,1) and CSO(7,1) gaugings

We analyze a particular G_2-invariant sector of the scalar manifold of gauged $\mathcal{N} = 8$ supergravity. The exact information on the supergravity potential implies a non-trivial operator algebra in dual field theory. From the effective scalar potential we have considered so far which consists of A_1 and A_2 tensors, one expects that the superpotential we are looking for maybe encoded in either A_1 or A_2 tensor. One can easily see that one of the eigenvalues of A_1 tensor, that is, $z_2(\lambda, \alpha)$ provides a superpotential which is related to the scalar potential of SO(7,1) or CSO(7,1) gauging as follows:

$$V(\lambda, \alpha) = \frac{16}{7} g^2 \left| \frac{\partial z_2}{\partial \lambda} \right|^2 - 6g^2 |z_2|^2\,, \tag{4.1}$$

where z_2 is given in (2.4) corresponding to SO(7,1) gauging or (3.1) corresponding to CSO(7,1) gauging. This coincides with the one corresponding to G_2 sector of compact SO(8) potential [7, 26]. The form of this scalar potential in terms of a superpotential is quite general for all the cases of SO(8), $SO(7,1)$ and CSO(7,1) gaugings. Although it seems to have there is no dependence on the derivative of z_2 with respect to the field α in the above (4.1), we have found that there exists an algebraic relation in complex z_2 field

$$\partial_\alpha \log |z_2| = 2\sqrt{2} pq \partial_\lambda \mathrm{Arg} z_2 \tag{4.2}$$

implying that one can write the derivative of z_2 respect to λ as two parts. We assume (2.5) here. Using this identity, that we have seen in compact SO(8) gauging also, we will arrive at the following relation with (2.5)

$$V(\lambda, \alpha) = \frac{16}{7} g^2 \left[\left(\frac{\partial W}{\partial \lambda} \right)^2 + \frac{1}{8p^2q^2} \left(\frac{\partial W}{\partial \alpha} \right)^2 \right] - 6g^2 W^2 \,, \qquad W = |z_2|$$

which is exactly the same as the one in G_2 sector of SO(8) potential. Contrary to the G_2 sector of compact SO(8) potential, one can check that there are no critical points for these sectors in SO(7,1) and CSO(7,1) potential by differentiating the superpotential W with respect to the λ and α fields.

The lagrangian of the scalar-gravity sector by adding the scalar potential we have found to the kinetic terms with vanishing A_μ^{IJ} can be obtained and is the same as the one in compact SO(8) gauging except the different potential. Then the resulting Lagrangian has the following form

$$\int d^4x \sqrt{-g} \left(\frac{1}{2} R - \frac{7}{8} \partial^\mu \lambda \partial_\mu \lambda - \frac{7}{4} s^2 \partial^\mu \alpha \partial_\mu \alpha - V(\lambda, \alpha) \right) \tag{4.3}$$

with (2.6) and $V(\lambda, \alpha)$ is a scalar potential for SO(7,1) gauging or CSO(7,1) gauging. To construct domain wall solution corresponding to the supergravity description of the nonconformal flow, the metric we are interested in is

$$ds^2 = e^{2A(r)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{2B(r)} dr^2 \,, \qquad \eta_{\mu\nu} = (-,+,+) \,.$$

With this ansatz it is straightforward to see that the equations of motion for the scalar and the metric from (4.3) are given

$$\begin{aligned}
2\partial_r^2 A + 3(\partial_r A)^2 - 2\partial_r A \partial_r B + \frac{7}{8} (\partial_r \lambda)^2 + \frac{7}{4} s^2 (\partial_r \alpha)^2 + e^{2B} V &= 0 \,, \\
\partial_r^2 \lambda + (3\partial_r A - \partial_r B)\, \partial_r \lambda - \sqrt{2} sc (\partial_r \alpha)^2 - \frac{4}{7} e^{2B} \partial_\lambda V &= 0 \,, \\
s^2 \partial_r^2 \alpha + s^2 (3\partial_r A - \partial_r B)\, \partial_r \alpha + \sqrt{2} sc \partial_r \lambda \partial_r \alpha - \frac{2}{7} e^{2B} \partial_\alpha V &= 0 \,.
\end{aligned} \tag{4.4}$$

Then the energy-density per unit area transverse to r-direction can be obtained. In order to get the first-order differential equations satisfying the domain-wall, we express the energy-density in terms of sum of complete sqaures. So one can find out the bound of the energy-density and it is extremized by the following domain-wall solutions. Note that in this

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derivation, we emphasize that the algebraic relation in (4.2) was crucial in order to cancel the unwanted terms. The flow equations with (2.5) are [7, 26]

$$\begin{aligned}
\partial_r \lambda(r) &= \pm \frac{8\sqrt{2}}{7}\, g\, e^{B(r)}\, \partial_\lambda W(\lambda,\alpha)\,, \\
\partial_r \alpha(r) &= \pm \frac{\sqrt{2}}{7p^2q^2}\, g\, e^{B(r)}\, \partial_\alpha W(\lambda,\alpha)\,, \\
\partial_r A(r) &= \mp \sqrt{2}\, g\, e^{B(r)}\, W(\lambda,\alpha)\,,
\end{aligned} \tag{4.5}$$

where $W = |z_2|$ and z_2 is given in (2.4) corresponding to $\mathrm{SO}(7,1)$ gauging or (3.1) corresponding to $\mathrm{CSO}(7,1)$ gauging. It is straightforward to verify that any solutions of $\lambda(r), \alpha(r)$ and $A(r)$ of (4.5) satisfy the gravitational and scalar equations of motion given by the second order equations (4.4). We have checked that there are no analytic solutions in (4.5).

4.2 SU(3) sectors of SO(6, 2) and CSO(6, 2) gaugings

We are looking for domain-wall solutions arising in supergravity theories with nontrivial superpotential defined on the restricted slice of the scalar manifold. By similar analysis, one gets the scalar potential and write it in terms of one of the eigenvalues of A_1 tensor

$$V(\lambda,\lambda',\alpha,\phi) = g^2\left[\frac{16}{3}\left|\frac{\partial z_3}{\partial\lambda}\right|^2 + 4\left|\frac{\partial z_3}{\partial\lambda'}\right|^2 - 6|z_3|^2\right].$$

Here z_3 is given in (2.7) for $\mathrm{SO}(6,2)$ gauging or (3.3) for $\mathrm{CSO}(6,2)$ gauging. This relation is coincident with the one of $\mathrm{SU}(3)$-invariant sector of $\mathrm{SO}(8)$ potential [7]. In other words, the above structure holds for $\mathrm{SO}(8), SO(6,2)$ and $\mathrm{CSO}(6,2)$ gaugings. At first sight, there are no λ' and ϕ-derivatives on the z_3. However, one can reexpress those dependences by introducing the absolute value of z_3 as the right superpotential. It is easy and straightforward to check that we have also two algebraic relations as follows:

$$\begin{aligned}
\partial_\alpha \log|z_3| &= 2\sqrt{2}pq\partial_\lambda \mathrm{Arg} z_3\,, \\
\partial_\phi \log|z_3| &= 2\sqrt{2}rt\partial_{\lambda'} \mathrm{Arg} z_3
\end{aligned} \tag{4.6}$$

providing that the derivative of z_3 with respect to λ can be decomposed into two parts and the one with respect to λ' into two parts. We assume also (2.5) and (2.8). Through these identities one can reexpress the above scalar potential as, together with (2.5) and (2.8),

$$\begin{aligned}
V(\lambda,\lambda',\alpha,\phi) &= g^2\left[\frac{16}{3}(\partial_\lambda W)^2 + \frac{2}{3p^2q^2}(\partial_\alpha W)^2 + 4(\partial_{\lambda'} W)^2 + \frac{1}{2r^2t^2}(\partial_\phi W)^2 - 6W^2\right], \\
W(\lambda,\lambda',\alpha,\phi) &= |z_3|\,.
\end{aligned}$$

The equations of motion for the scalar and the metric are given

$$\begin{aligned}
2\partial_r^2 A + (3\partial_r A - 2\partial_r B)\partial_r A + \frac{3}{8}(\partial_r\lambda)^2 + \frac{3}{4}s^2(\partial_r\alpha)^2 + \frac{1}{2}(\partial_r\lambda')^2 + s'^2(\partial_r\phi)^2 + e^{2B}V &= 0\,, \\
\partial_r^2\lambda + (3\partial_r A - \partial_r B)\partial_r\lambda - \sqrt{2}sc(\partial_r\alpha)^2 - \frac{4}{3}e^{2B}\partial_\lambda V &= 0\,,
\end{aligned}$$

$$\begin{aligned}
\partial_r^2\lambda' + (3\partial_r A - \partial_r B)\partial_r\lambda' - \sqrt{2}s'c'(\partial_r\phi)^2 - e^{2B}\partial_{\lambda'}V &= 0\,,\\
s^2\partial_r^2\alpha + s^2(3\partial_r A - \partial_r B)\partial_r\alpha + \sqrt{2}sc\partial_r\alpha\partial_r\lambda - \frac{2}{3}e^{2B}\partial_\alpha V &= 0\,,\\
s'^2\partial_r^2\phi + s'^2(3\partial_r A - \partial_r B)\partial_r\phi + \sqrt{2}s'c'\partial_r\phi\partial_r\lambda' - \frac{1}{2}e^{2B}\partial_\phi V &= 0\,.
\end{aligned} \tag{4.7}$$

For given Lagrangian where the kinetic terms are the same as the one in the compact SO(8) gauging and given in [7], the energy-density can be obtained with the domain wall ansatz we have considered. In this case, by using the two relations in (4.6) the flow equations [7] with (2.5) and (2.8) are

$$\begin{aligned}
\partial_r\lambda(r) &= \pm\frac{8\sqrt{2}}{3}g\, e^{B(r)}\,\partial_\lambda W(\lambda,\lambda',\alpha,\phi)\,,\\
\partial_r\lambda'(r) &= \pm 2\sqrt{2}g\, e^{B}(r)\,\partial_{\lambda'} W(\lambda,\lambda',\alpha,\phi)\,,\\
\partial_r\alpha(r) &= \pm\frac{\sqrt{2}}{3p^2q^2}g\, e^{B(r)}\,\partial_\alpha W(\lambda,\lambda',\alpha,\phi)\,,\\
\partial_r\phi(r) &= \pm\frac{\sqrt{2}}{4r^2t^2}g\, e^{B(r)}\,\partial_\phi W(\lambda,\lambda',\alpha,\phi)\,,\\
\partial_r A(r) &= \mp\sqrt{2}g\, e^{B(r)}\, W(\lambda,\lambda',\alpha,\phi)\,,
\end{aligned} \tag{4.8}$$

where $W = |z_3|$ and we put the $B(r)$ dependence in the right hand side and z_3 is given in (2.7) for SO(6, 2) gauging or (3.3) for CSO(6, 2) gauging. Any solutions of $\lambda(r), \lambda'(r), \alpha(r), \phi(r)$ and $A(r)$ of (4.8) satisfy the gravitational and scalar equations of motion given by the second order equations (4.7). There are no analytic solutions in (4.8).

It is easy to see that the scalar potential for SO(5) sectors of SO(5, 3) and CSO(5, 3) gaugings can be expressed in terms of a superpotential as follows

$$\begin{aligned}
V(\lambda,\mu,\rho) = g^2\Bigg[&\frac{32}{5}(\partial_\lambda W)^2 + \frac{32}{5}(\partial_\mu W)^2 + \frac{32}{5}(\partial_\rho W)^2 - \\
&- \frac{16}{5}\partial_\lambda W\partial_\mu W - \frac{16}{5}\partial_\lambda W\partial_\rho W - \frac{16}{5}\partial_\mu W\partial_\rho W - 6W^2\Bigg]\,,
\end{aligned} \tag{4.9}$$

where $W = z_1$ is a superpotential (2.12) for SO(5, 3) gauging or (3.9) for CSO(5, 3) gauging. The form of this scalar potential in terms of a superpotential is quite general for all the cases of SO(8), SO(5, 3) and CSO(5, 3) gaugings. For general λ, μ, ρ, due to the mixed terms in the above (4.9) and in the kinetic terms [7], there are no domain wall solutions for SO(5, 3) gauging but under the subspace $\lambda = \mu = \rho$ we have seen that there is a BPS domain solution [1] for SO(5, 3) gauging.

5. The potentials of CSO$(p, q, 8-p-q)$ gauged supergravity

According to the result of [1], the scalar potential of CSO$(p, q, 8-p-q)$ gauging which is invariant subspace under a particular SO(p) × SO(q) × SO$(8-p-q)$ subgroup of SO(8) can be read off. The CSO$(p, q, 8-p-q)$ gauging and the CSO$(q, p, 8-p-q)$ gauging are equivalent to each other. So we describe half of them here.

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CSO($p, 6-p, 2$) gaugings ($p = 3, 4, 5$). Let us consider the scalar potential of CSO(3, 3, 2) gauging given in terms of two real scalar fields $\widetilde{m}, \widetilde{n}$ by putting $\xi = -1$, $\zeta = 0$, and $p = 3 = q$ in the general form of scalar potential of CSO($p, q, 8-p-q$) gauging [1]. It is given by

$$V_{3,3,2} = -\frac{3}{4} g^2 e^{-\sqrt{\frac{2}{3}}\widetilde{n}} \left[\cosh(2\lambda) \mp 3\right] , \qquad \widetilde{m} \to \sqrt{\frac{3}{2}}\lambda ,$$

where the + sign in the last term in the above means the CSO(6, 2) gauging because in this case, $\xi = 1$ and $\zeta = 0$. At the subspace of $\widetilde{n} = 0$, the CSO(3, 3, 2) potential is proportional to the scalar potential of noncompact SO(3, 3) gauging in five dimensional supergravity [11]. This theory in five dimensions has a de Sitter critical point at which the scalar λ vanishes and there is no supersymmetry. Then the scalar potential which is a SO(3) × SO(3) invariant sector of CSO(3, 3, 2) gauging becomes

$$V = \frac{3}{2} g^2 , \qquad \text{at} \qquad \lambda = 0 . \tag{5.1}$$

On the other hand, for $\xi = 1, \zeta = 0$, when the λ vanishes, the scalar potential with $\widetilde{n} = 0$ has $V = -3g^2$ we have discussed before.[6] In subsection 3.2, we have seen the scalar potential in terms of two fields, λ' and ϕ. This critical point at which $V = -3g^2$ corresponds to SO(3)×SO(3) invariant critical point in compact SO(6) gauged supergravity in five dimensions. The potential $V_{3,3,2}$ has the exponential roll in the $\widetilde{n}$ direction but is unbounded below in the λ direction [16]. Note that we have found that in [1] there exists an analytic solution for domain wall of CSO(3, 3, 2) gauging.

Similarly the scalar potential of CSO(4, 2, 2) gauging by putting $\xi = -1$, $\zeta = 0$, and $p = 4, q = 2$ in the general form of scalar potential of CSO($p, q, 8-p-q$) gauging is given by

$$V_{4,2,2} = -g^2 e^{-\sqrt{\frac{2}{3}}\widetilde{n}} \left(e^{2\lambda} \mp 2e^{-\lambda}\right) , \qquad \widetilde{m} \to \sqrt{3}\lambda ,$$

where the + sign in the last term in the above means the CSO(6, 2) gauging because in this case, $\xi = 1$ and $\zeta = 0$. At the subspace of $\widetilde{n} = 0$, the CSO(4, 2, 2) potential is proportional to the scalar potential of SO(4, 2) gauging in five dimensional supergravity [11] which has no critical points. The SO(4)×SO(2) invariant scalars of the SO(6) gauging in 5-dimensions lead to no new critical points. The scalar potential gives $V = -3g^2$ at $\lambda = 0$ (in this case, $\xi = 1$ and $\zeta = 0$).

Finally, the scalar potential of CSO(5, 1, 2) gauging by putting $\xi = -1$, $\zeta = 0$, and $p = 5, q = 1$ in the general form of CSO($p, q, 8-p-q$) gauging is given by

$$V_{5,1,2} = -\frac{1}{8} g^2 e^{-\sqrt{\frac{2}{3}}\widetilde{n}} \left(15e^{2\lambda} \mp 10e^{-4\lambda} - e^{-10\lambda}\right) , \qquad \widetilde{m} \to \sqrt{\frac{15}{2}}\lambda ,$$

where the + sign in the last term means the CSO(6, 2) gauging because in this case, $\xi = 1$ and $\zeta = 0$. At the subspace of $\widetilde{n} = 0$, the CSO(5, 1, 2) potential is proportional to the scalar potential of SO(5, 1) gauging in five dimensions [11] which has no critical points.

[6] The CSO(6, 2) scalar potential is $V_{6,2} = -3g^2 e^{2s}$ [5] where s is a scalar field that is proportional to the above $\widetilde{n}$.

The SO(5) invariant scalar of the SO(6) gauging in 5-dimensions($\xi = 1$ and $\zeta = 0$) leads to the scalar potential with $\widetilde{n} = 0$

$$V = -\frac{1}{2} \times 3^{5/3} g^2 \qquad \text{at} \qquad \lambda = -\frac{1}{6} \log 3 \,. \tag{5.2}$$

Although there are no direct relations between the supergravity potentials in four dimensions and in five dimensions, the observation that the CSO(6,2) scalar potential in four dimensions is related to the scalar potential for SO(5) sector of the SO(6) gauging in five dimensions will provide some hints to understand the structure of five dimensional scalar potential in the context of full scalar manifold. The existence of an unstable nonsupersymmetric SO(5)-invariant background of $AdS_5 \times \mathbf{S}^5$ of type IIB string theory was studied in [12, 13] from the mass spectrum of the low-lying states in this SO(5)-invariant supergravity solution. Moreover, the scalar potential becomes $V = -3g^2$ at $\lambda = 0$ ($\xi = 1$ and $\zeta = 0$) which is common to the $\mathrm{CSO}(p, 6-p, 2)$ gaugings ($p = 3, 4, 5$). We expect that the $\mathrm{SO}(p, 6-p)$[SO(6)] gauge theories in five dimensions reduce to the $\mathrm{CSO}(p, 6-p, 2)$[CSO(6,2)] gauge theories in four dimensions.

$\mathrm{CSO}(p, 5-p, 3)$ gaugings ($p = 3, 4$). Let us analyze the scalar potential of CSO(3,2,3) gauging given in terms of two real scalar fields $\widetilde{m}, \widetilde{n}$ by putting $\xi = -1$, $\zeta = 0$, and $p = 3, q = 2$ in the general form of scalar potential. The potential reads

$$V_{3,2,3} = -\frac{3}{8} g^2 e^{-\sqrt{\frac{6}{5}}\widetilde{n}} \left(e^{4\lambda} \mp 4 e^{-\lambda} \right), \qquad \widetilde{m} \to \sqrt{\frac{15}{2}} \lambda \,,$$

where one takes $-$ for the CSO(3,2,3) in the second term and $+$ for the CSO(5,3) theories. At the subspace of $\widetilde{n} = 0$, the CSO(3,2,3) potential is proportional to the scalar potential of SO(3,2) gauging in seven dimensional gauged supergravity [17]. This theory in seven dimensions has no critical point. On the other hand, the SO(3) × SO(2) invariant scalar of the SO(5) gauging ($\xi = 1$ and $\zeta = 0$) in seven dimensions leads to the scalar potential $V = -\frac{15}{8} g^2$ at $\lambda = 0$.[7] In subsection 3.3, we have seen the scalar potential in terms of λ, μ and ρ. At $\lambda = \mu = \rho = 0$, the scalar potential will coincide with this value.

Similarly the scalar potential of CSO(4,1,3) gauging by putting $\xi = -1$, $\zeta = 0$, and $p = 4, q = 1$ in the general form of scalar potential is given by

$$V_{4,1,3} = -\frac{1}{8} g^2 e^{-\sqrt{\frac{6}{5}}\widetilde{n}} \left(8 e^{2\lambda} - e^{-8\lambda} \mp 8 e^{-3\lambda} \right), \qquad \widetilde{m} \to \sqrt{5} \lambda \,.$$

For $-$ sign in the last term for CSO(4,1,3) gauging equivalent to SO(4,1) gauged theory in seven dimensions, there is no critical point. In seven dimensional gauged supergravity side, they exist a local maximum for $\lambda = 0$(at which $V = -\frac{15}{8} g^2$) possessing stable and maximally supersymmetric SO(5) symmetry and a local minimum for $\lambda = -\frac{1}{5} \log 2$ with unstable nonsupersymmetric SO(4) symmetry. The scalar potential gives

$$V = -\frac{5}{4} \times 2^{3/5} g^2 \,, \qquad \text{at} \qquad \lambda = -\frac{1}{5} \log 2 \,. \tag{5.3}$$

Summarizing we expect that the $\mathrm{SO}(p, 5-p)$[SO(5)] gauge theories in seven dimensions reduce to the $\mathrm{CSO}(p, 5-p, 3)$[CSO(5,3)] gauge theories in four dimensions.

[7] The CSO(5,3) scalar potential [5] is $V_{5,3} = -\frac{15}{8} g^2 e^{2s}$ where s is a scalar field that is proportional to the above $\widetilde{n}$.

CSO($p,4-p,4$) gaugings ($p=2,3$). The scalar potential of CSO(2,2,4) gauging by putting $\xi=-1$, $\zeta=0$, and $p=2=q$ in the general form of scalar potential is

$$V_{2,2,4}=\pm g^2 e^{\phi/2}\,,\qquad \widetilde{n}\to-\frac{\phi}{2\sqrt{2}}\,.$$

For $-$ sign in the above it is equivalent to CSO(4,4) gauged theory. At the subspace of $\phi=0$, since the potential has a constant value, it is a critical point of SO(2) $\times$ SO(2) sector of CSO(4,4) gauging ($\xi=1$ and $\zeta=0$). On the other hand, one can interprete the + sign in the above as a noncompact SO(2,2) gauged supergravity in seven dimensions that is a noncompact version of compact SO(4) gauging. The scalar potential of compact SO(4) gauged supergravity was constructed in [18]. By taking the appropriate SO(2,2) metric for T-tensor T_{ij}, it is easy to see that one gets the above potential.

Similarly the scalar potential of CSO(3,1,4) gauging is given by

$$V_{3,1,4}=-\frac{1}{8}g^2 e^{\phi/2}\left(3e^{2\lambda}\mp 6e^{-2\lambda}-e^{-6\lambda}\right),\qquad \widetilde{m}\to\sqrt{3}\lambda\,,\qquad \widetilde{n}\to-\frac{\phi}{2\sqrt{2}}\,,$$

where one takes $-$ for the CSO(3,1,4) in the second term and + for the CSO(4,4) theories. The former is proportional to the scalar potential of SO(3,1) gauging in seven dimensions [18]. This theory in seven dimensions has no critical point. On the other hand, the SO(3) invariant scalar of the SO(4) gauging in seven dimensional supergravity leads to the scalar potential with $\phi=0$

$$V=-g^2\,,\qquad \text{at}\qquad \lambda=0\,. \tag{5.4}$$

Recall that the CSO(4,4) scalar potential reads $V_{4,4}=-g^2e^{2s}$ where s is a scalar field [5]. We expect that the SO($p,4-p$)[SO(4)] gauge theories in seven dimensions reduce to the CSO($p,4-p,4$)[CSO(4,4)] gauge theories in four dimensions.

CSO(2,1,5) gauging. Let us consider the scalar potential of CSO(2,1,5) gauging given in terms of two real scalar fields $\widetilde{m},\widetilde{n}$ by putting $\xi=-1$, $\zeta=0$, and $p=2,q=1$ in the general form of scalar potential. It is given by

$$V_{2,1,5}=\frac{1}{8}g^2 e^{-2\phi}\left(\pm e^{-8\lambda}+4e^{-2\lambda}\right),\qquad \widetilde{m}\to\sqrt{6}\lambda,\qquad \widetilde{n}\to\frac{\sqrt{6}\phi}{\sqrt{5}}\,,$$

where the $-$ sign in the first term in the above means the CSO(3,5) gauging because in this case, $\xi=1$ and $\zeta=0$. The above CSO(2,1,5) scalar potential is proportional to the scalar potential of noncompact SO(2,1) gauging in eight dimensions, that is a noncompact version of compact SO(3) gauging [19, 20], obtained by taking the appropriate SO(2,1) metric for T-tensor T_{ij}. This theory in eight dimensions has no critical point. On the other hand, the SO(2) invariant scalar of the SO(3) gauging in eight dimensions leads to the scalar potential(with $\phi=0$) $V=-\frac{3}{8}g^2$ at $\lambda=0$ ($\xi=1$ and $\zeta=0$). The CSO(3,5) scalar potential [5] was $V_{3,5}=-\frac{3}{8}g^2e^{2s}$ where s is a scalar field. We expect that the SO(2,1)[SO(3)] gauge theories in eight dimensions reduce to the CSO(2,1,5)[CSO(3,5)] gauge theories in four dimensions.

CSO(1, 1, 6) gauging. The scalar potential of CSO(1, 1, 6) gauging by putting $\xi = -1$, $\zeta = 0$, and $p = 1 = q$ in the general form of scalar potential is given by

$$V_{1,1,6} = \frac{1}{8} g^2 e^{\frac{4}{\sqrt{7}}\phi} \left(e^{2\lambda} + e^{-2\lambda} \pm 2 \right), \qquad \widetilde{m} \to \frac{1}{\sqrt{2}}\lambda, \qquad \widetilde{n} \to -\frac{4\phi}{\sqrt{42}},$$

where one takes + for the CSO(1, 1, 6) in the last term and − for the CSO(2, 6) theories. By taking the appropriate SO(1, 1) metric for T-tensor T_{ij}, the former is proportional to the scalar potential of noncompact SO(1, 1) gauging in nine dimensions which is a noncompact version of compact SO(2) gauging [21, 22, 15]. The SO(1) invariant scalar of the SO(2) gauging in nine dimensions leads to one critical point. The scalar potential gives $V = 0$ at $\lambda = 0(\xi = 1$ and $\zeta = 0)$ corresponding to CSO(2, 6) theory. The CSO(2, 6) scalar potential [5] was $V_{2,6} = 0$. Note that the exponential dependence on ϕ implies that $V_{1,1,6}$ can have only critical points at values $\lambda = \lambda_0$ which are critical points of $(e^{2\lambda} + e^{-2\lambda} - 2)$(the derivative of this with respect to λ should vanish at $\lambda = \lambda_0$), at which the potential vanishes. In this case the full potential restricted to the scalar manifold parametrized by both ϕ and λ has a critical point at $\lambda = 0$. Note that there exists an analytic domain wall solution for this case [1]. The SO(1, 1)[SO(2)] gauge theories in nine dimensions reduce to the CSO(1, 1, 6)[CSO(2, 6)] gauge theories in four dimensions.

CSO(p, 7−p, 1) gaugings (p = 4, 5, 6). Let us consider the scalar potential of CSO(4, 3, 1) gauging by putting $\xi = -1$, $\zeta = 0$, and $p = 4, q = 3$ in the general form of scalar potential and it is given by

$$V_{4,3,1} = -\frac{1}{8} g^2 e^{\frac{2}{7}\phi} \left(8e^{2\lambda} \mp 24 e^{-\frac{\lambda}{3}} + 3e^{-\frac{8\lambda}{3}} \right), \qquad \widetilde{m} \to \frac{\sqrt{21}}{3}\lambda, \qquad \widetilde{n} \to -\sqrt{\frac{2}{7}}\phi,$$

where the + sign in the last term in the above means the CSO(7, 1) gauging because in this case, $\xi = 1$ and $\zeta = 0$. This CSO(4, 3, 1) potential is proportional to the scalar potential of SO(4, 3) gauging in four dimensions. This theory has the scalar potential(with $\phi = 0$) is given by, in the SO(4) × SO(3) invariant sector of CSO(4, 3, 1) gauging,

$$V = \frac{7}{8} \times 2^{8/7} g^2 \qquad \text{at} \qquad \lambda = -\frac{3}{7} \log 2 \,. \tag{5.5}$$

On the other hand, for $\xi = 1, \zeta = 0$ there is a scalar potential with $\phi = 0$ which has $V = -\frac{35}{8} g^2$ we have discussed before(The CSO(7, 1) scalar potential [5] was $V_{7,1} = -\frac{35}{8} g^2 e^{2s}$ where s is a scalar field). In subsection 3.1, we have seen the scalar potential $V(\lambda)$ at the $\alpha = 0$. At $\lambda = 0$, that potential becomes the same cosmological constant, $V = -\frac{35}{8} g^2$.

Similarly the scalar potential of CSO(5, 2, 1) gauging is given by

$$V_{5,2,1} = \frac{5}{8} g^2 e^{\frac{2}{7}\phi} \left(-3e^{2\lambda} \mp 4e^{-\frac{3\lambda}{2}} \right), \qquad \widetilde{m} \to \sqrt{\frac{35}{8}}\lambda, \qquad \widetilde{n} \to -\sqrt{\frac{2}{7}}\phi,$$

where one takes − for the CSO(5, 2, 1) in the second term and + for the CSO(7, 1) theories. The CSO(5, 2, 1) potential is proportional to the scalar potential of SO(5, 2) gauging in four dimensions. This theory has no critical point. On the other hand, the SO(5) × SO(2) invariant scalar of the SO(7) gauging in four dimensions($\xi = 1$ and $\zeta = 0$) leads to the scalar potential(with $\phi = 0$) $V = -\frac{35}{8} g^2$ at $\lambda = 0$.

Finally the scalar potential of CSO(6,1,1) gauging by putting $\xi = -1$, $\zeta = 0$, and $p = 6, q = 1$ in the general form of scalar potential is given by

$$V_{6,1,1} = \frac{1}{8} g^2 e^{\frac{2}{7}\phi} \left(-24 e^{2\lambda} \mp 12 e^{-5\lambda} + e^{-12\lambda} \right), \qquad \widetilde{m} \to \sqrt{\frac{21}{2}} \lambda \,, \qquad \widetilde{n} \to -\sqrt{\frac{2}{7}} \phi \,,$$

where one takes $-$ for the CSO(6,1,1) in the second term and $+$ for the CSO(7,1) theories. The former is proportional to the scalar potential of SO(6,1) gauging in four dimensions. This theory has no critical point. On the other hand, the SO(6) invariant scalar of the SO(7) gauging in four dimensions leads to the scalar potential $V = -\frac{35}{8} g^2$ at $\lambda = 0 (\xi = 1$ and $\zeta = 0)$ and for SO(6) invariant sector of CSO(7,1) gauging in four dimensions

$$V = -7 \times 2^{-4/7} g^2 \qquad \text{at} \qquad \lambda = -\frac{1}{7} \log 4 \,. \tag{5.6}$$

In this case, the SO$(p, 7-p)$[SO(7)] gauge theories in four dimensions are related to the CSO$(p, 7-p, 1)$[CSO(7,1)] gauge theories in four dimensions.

6. Discussions

In summary,

- In section 2, we constructed a superpotential from A_1 tensor for G_2 sector for SO(7,1) gauging, SU(3) sector for SO(6,2) gauging, SO(5) sector for SO(5,3) gauging and SO(3) $\times$ SO(3) sector for SO(4,4) gauging. In particular, the superpotentials are the magnitudes of z_2 in (2.4) for SO(7,1) gauging or (3.1) for CSO(7,1) gauging while they are given as the magnitudes of z_3 in (2.7) for SO(6,2) gauging or (3.3) for CSO(6,2) gauging. All these provide the first order differential equations.

- In section 3, we generalized to the G_2 sector for CSO(7,1) gauging, SU(3) sector for CSO(6,2) gauging, SO(5) sector for CSO(5,3) gauging. Specially, we have discovered three new AdS critical points characterized by (3.7) in the SU(3) sector for CSO(6,2) gauging, in the four parameter space of full scalar manifold, preserving the SU(3)-invariance. When we restrict to the subspace parametrized by λ' only, the scalar potential shows the one in the SU(3)-invariant sector of compact SO(6) gauged supergravity in five-dimensions.

- In section 4, we obtained the first order domain wall solutions for G_2 sectors (4.5) for SO(7,1) and CSO(7,1) gaugings and SU(3) sectors (4.8) for SO(6,2) and CSO(6,2) gaugings by rewriting the scalar potential in terms of a superpotential. The observation of (4.2) and (4.6) played the role of elliminating the terms we do not want in the energy-functional.

- In section 5, we analyzed the behavior of the scalar potentials in the CSO$(p, q, 8-p-q)$ gauged supergravity theory. Along the line of the critical points we have found newly in section 3, the potential characterized by (5.2) in the SO(5) sector for CSO(6,2) gauging was exactly the scalar potential for SO(5)-invariant sector of compact SO(6) gauged supergravity in five-dimensions. Also we realized that CSO(3, 3, 2) gauging is given in (5.1) and it implies the potential for SO(3) × SO(3)-invariant sector of noncompact SO(3, 3) gauged supergravity in five-dimensions. There exists a potential (5.3) in the SO(4) sector for CSO(5, 3) gauging in the reduced parameter space corresponding to SO(4)-invariant sector of the compact SO(5) gauged supergravity in seven-dimensions. We have obtained the potential (5.5) in the SO(4) × SO(3) sector for CSO(4, 3, 1) gauging in the reduced parameter space corresponding to SO(4) × SO(3)-invariant sector of the compact SO(7) gauged supergravity in four-dimensions. There exists (5.4) in the SO(3) sector for CSO(4, 4) gauging in the reduced parameter space corresponding to SO(3)-invariant sector of the compact SO(4) gauged supergravity in seven-dimensions. Also there was (5.6) in the SO(6) sector for CSO(7, 1) gauging corresponding to the SO(6) invariant sector of the SO(7) gauged supergravity in four dimensions.

Let us describe the future directions. The scalar potential of gauged $\mathcal{N} = 8$ supergravity in four dimensions is a function of 70 scalars. We can reduce the problem by searching for all critical points that reduce the gauge/R-symmetry to a group containing a particular SO(3) subgroup of SO(8). It is known that all of the 35-dimensional representations of SO(8) contain three SU(3)-singlets. That is $\mathbf{8} + \mathbf{6} + \bar{\mathbf{6}} + \mathbf{3} + \mathbf{3} + \bar{\mathbf{3}} + \bar{\mathbf{3}} + \mathbf{1} + \mathbf{1} + \mathbf{1}$. Under the SO(3) subgroup of SU(3), the irreducible representation $\mathbf{6}$ of SU(3) breaks into $\mathbf{5} + \mathbf{1}$. Therefore, SO(3)-singlet space with a breaking of the SO(8) gauge group into a group which contains SO(3) may be parametrized by ten real fields. We expect that there will be new critical points in the SO(3) sector of guaged $\mathcal{N} = 8$ supergravity in four dimensions. In the context of present work, the 28-beins in those sector can be used in the SO(3) sector of CSO(6, 2) gauged supergravity. At least one should find out two AdS critical points. At nonsupersymmetric critical point, the potential gives $V = -\frac{3}{2}(\frac{25}{2})^{1/3}g^2$ corresponding to $\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)$ gauge symmetry in the five dimensional supergravity and at nonsupersymmetric critical point, the potential will be $V = -\frac{2^{10/3}}{3}g^2$ corresponding to $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge symmetry in the supergravity side [23]. Note that the critical points in the SU(2) sector of gauged supergravity in five dimensions are exactly the same as the one in the SO(3) sector in that theory [24]. Similarly, it would be interesting to study SU(2)-singlet space with a breaking of the SO(8) gauge group into a group which contains SU(2). Among the possible branching rules of $\mathbf{3}, \bar{\mathbf{3}}, \mathbf{6}, \mathbf{8}$ into the representations of SU(2), the largest singlet structure in $E_{7(7)}$ will provide new critical points.

When one reduces 11-dimensional supergravity theory to four dimensional $\mathcal{N} = 8$ supergravity, the four dimensional spacetime is warped by warp factor that provides an understanding of the different scales of the 11-dimensional solutions. The nonlinear metric ansatz in [8] provides the explicit formula for the 7-dimensional inverse metric that is encoded by the warp factor, Killing vectors and 28-beins in four dimensional gauged supergravity theory. In part I, we identified the 28-beins for $\mathrm{SO}(p) \times \mathrm{SO}(8-p)$ sectors with a single vacuum expectation value ϕ which depends on the AdS_4 radial coordinate r. With the insertion of ξ-dependence in the u, v, one can easily see that the general expressions for u, v can be obtained by simply replacing ϕ with $(\phi - t)$ because our u, v are related to

$\mathcal{V}E^{-1}(t)$ and we do not need any Baker-Hausdorff formula. As we have done in the compact gauged supergravity [25]–[27], one introduces the standard metric of a 7-dimensional ellipsoid characterized by the following diagonal matrix $Q_{AB} = \text{diag}\left(\mathbf{1}_p, \xi e^{-(1+\beta)\phi}\mathbf{1}_{8-p}\right)$, where $\beta = p/(8-p)$. Then the 7-dimensional metric can be written as $dX^A Q_{AB}^{-1} dX^B$ where the $\mathbb{R}^8$ coordinate $X^A (A = 1, \ldots, 8)$ are constrained on the unit round 7-sphere, $\sum_A (X^A)^2 = 1$. Note that the quadratic form $\Xi^2 = X^A Q_{AB} X^B$ turns to 1 for the round 7-sphere with $\phi = 0$ and $\xi = 1$. The warp factor introduced in [28, 29] is nothing but our Ξ^2. Applying the Killing vectors together with the 28-beins u, v to the metric formula, with the multiplication of e^{-2t}, one obtains an inverse metric including the warp factor. However, in order to get the full 7-dimensional metric, one has to separate out the warp factor from those results. By plugging the metric with warp factor into the definition of warp factor, one gets a self-consistent equation for warp factor. With this explicit form of warp factor, we will get the final full warped 7-dimensional metric corresponding to the one obtained in [28].

With the insertion of ξ, ζ-dependence in the u, v for $\text{CSO}(p, q, 8-p-q)$ gauging, one can see that the general expressions for u, v can be obtained because our u, v are related to $\mathcal{V}E^{-1}(t) \times \mathcal{V}E^{-1}(s)$ and we do not need any Baker-Hausdorff formula. Therefore we replace ϕ with $(\phi - t)$ and χ with $(\chi - s)$. One introduces the standard metric of a 7-dimensional ellipsoid characterized by the following diagonal matrix $Q_{AB} = \text{diag}(\mathbf{1}_p, \xi e^{-(1+\beta)\phi}\mathbf{1}_q, \xi\zeta e^{-(1+\beta)\phi} e^{-(1+\beta')\chi}\mathbf{1}_{8-p-q})$ where $\beta = p/(8-p)$ and $\beta' = (p+q)/(8-p-q)$. Then the 7-dimensional metric can be written as $dX^A Q_{AB}^{-1} dX^B$. Note that the quadratic form $\Xi^2 = X^A Q_{AB} X^B$ turns to 1 for the round 7-sphere with $\phi = 0 = \chi$ and $\xi = 1 = \zeta$. The warp factor introduced in [28, 29] is nothing but our Ξ^2. For G_2 sector for $\text{SO}(7,1)$ gauging, $\text{SU}(3)$ sector for $\text{SO}(6,2)$ gauging, $\text{SO}(5)$ sector for $\text{SO}(5,3)$ gauging and $\text{SO}(3) \times \text{SO}(3)$ sector for $\text{SO}(4,4)$ gauging, it would be interesting to develop the full warped 7-dimensional metric.

It is natural to ask whether 11-dimensional embedding of various vacua we have considered of non-compact and non-semi-simple gauged supergravity can be obtained. In [28], the metric on the 7-dimensional internal space and domain wall in 11-dimensions was found. However, an ansatz for an 11-dimensional three-form gauge field is still missing. It would be interesting to study the geometric superpotential, 11-dimensional analog of superpotential we have obtained. We expect that the nontrivial r-dependence of vevs makes Einstein-Maxwell equations consistent not only at the critical points but also along the RG flow connecting two critical points.

A. Nonzero A_2 tensors for given sectors of gauged supergravities

The nonzero components of A_2 tensors can be obtained from (2.2) and (2.1) by inserting the 28-beins u, v given in the [7, appendix A or B] and the projectors in the [1, appendix F]. For $\text{SO}(p, 8-p)$ gauging we put $\xi = -1$ and for $\text{CSO}(p, 8-p)$ gauging we have $\xi = 0$. Now we classify them below.

G_2 sector of $\text{SO}(7,1)$ gauging. In this case, the components of A_2 tensor $A_{2,l}^{\;\;ijk}$ can be represented by three different fields $y_{i,-} (i = 1, 2, 3)$ with degeneracies 7,21,28 respectively and given by

$$\begin{aligned}
{A_{2,8}}^{172} &= {A_{2,8}}^{163} = {A_{2,8}}^{154} = {A_{2,8}}^{253} = {A_{2,8}}^{246} = {A_{2,8}}^{374} = {A_{2,8}}^{576} \equiv y_{1,-} \\
{A_{2,1}}^{278} &= {A_{2,1}}^{368} = {A_{2,1}}^{458} = {A_{2,2}}^{187} = {A_{2,2}}^{358} = {A_{2,2}}^{486} = {A_{2,3}}^{186} = {A_{2,3}}^{285} = {A_{2,3}}^{478} \\
&= {A_{2,4}}^{185} = {A_{2,4}}^{268} = {A_{2,4}}^{387} = {A_{2,5}}^{148} = {A_{2,5}}^{238} = {A_{2,5}}^{678} = {A_{2,6}}^{138} = {A_{2,6}}^{284} \\
&= {A_{2,6}}^{587} = {A_{2,7}}^{128} = {A_{2,7}}^{348} = {A_{2,7}}^{568} \equiv y_{2,-} \\
{A_{2,1}}^{234} &= {A_{2,1}}^{256} = {A_{2,1}}^{375} = {A_{2,1}}^{467} = {A_{2,2}}^{143} = {A_{2,2}}^{165} = {A_{2,2}}^{367} = {A_{2,2}}^{457} = {A_{2,3}}^{124} \\
&= {A_{2,3}}^{157} = {A_{2,3}}^{276} = {A_{2,3}}^{456} = {A_{2,4}}^{132} = {A_{2,4}}^{176} = {A_{2,4}}^{275} = {A_{2,4}}^{365} = {A_{2,5}}^{173} \\
&= {A_{2,5}}^{126} = {A_{2,5}}^{247} = {A_{2,5}}^{346} = {A_{2,6}}^{152} = {A_{2,6}}^{147} = {A_{2,6}}^{237} = {A_{2,6}}^{354} = {A_{2,7}}^{164} \\
&= {A_{2,7}}^{135} = {A_{2,7}}^{263} = {A_{2,7}}^{254} \equiv y_{3,-}
\end{aligned} \tag{A.1}$$

where their explicit forms are

$$\begin{aligned}
y_{1,-} &= \frac{1}{4}\, e^{-i\alpha} \left(p + e^{i\alpha} q\right)^2 \Big[-3p^4 q - 3e^{5i\alpha} p q^4 + e^{4i\alpha} q^3 \left(12p^2 + q^2\right) + e^{i\alpha} p^3 \left(p^2 + 12q^2\right) + \\
&\qquad + e^{2i\alpha} \left(4p^4 q - 6p^2 q^3\right) + e^{3i\alpha} \left(-6p^3 q^2 + 4pq^4\right)\Big], \\
y_{2,-} &= -\frac{1}{4}\, e^{-5i\alpha} \left(e^{i\alpha} p + q\right) \Big[3e^{6i\alpha} p^4 q^2 + 4e^{3i\alpha} p^3 q^3 + 3p^2 q^4 - 2e^{i\alpha} p q^3 \left(4p^2 + q^2\right) - \\
&\qquad - 2e^{5i\alpha} p^3 q \left(p^2 + 4q^2\right) - e^{2i\alpha} q^2 \left(2p^4 + 8p^2 q^2 + q^4\right) - \\
&\qquad - e^{4i\alpha} p^2 \left(p^4 + 8p^2 q^2 + 2q^4\right)\Big], \\
y_{3,-} &= \frac{1}{4}\, e^{-3i\alpha} \Big[-3p^4 q^3 - 3e^{7i\alpha} p^3 q^4 + e^{6i\alpha} p^2 q^3 \left(4p^2 + 3q^2\right) + e^{i\alpha} p^3 q^2 \left(3p^2 + 4q^2\right) + \\
&\qquad + e^{4i\alpha} q^3 \left(6p^4 + q^4\right) + 3e^{5i\alpha} p q^2 \left(2p^4 + 4p^2 q^2 + q^4\right) + \\
&\qquad + 3e^{2i\alpha} p^2 q \left(p^4 + 4p^2 q^2 + 2q^4\right) + e^{3i\alpha} p^3 \left(p^4 + 6q^4\right)\Big]
\end{aligned} \tag{A.2}$$

together with (2.5). It is clear that ${A_{2,l}}^{ijk} = -{A_{2,l}}^{ikj}$ and ${A_{2,l}}^{ijk} = {A_{2,l}}^{jki} = {A_{2,l}}^{kij}$.

G_2 sector of CSO(7, 1) gauging. With $\xi = 0$, they are classified by three different fields $y_{i,0} (i = 1, 2, 3)$ with degeneracies 7,21,28 respectively and given by (A.1) with the replacement $y_{i,-} \to y_{i,0}$. The redefined expressions are

$$\begin{aligned}
y_{1,0} &= \frac{1}{8}\, e^{-i\alpha} \left(p - e^{i\alpha} q\right)^3 \left(p + e^{i\alpha} q\right)^2 \left(-7pq + 7e^{2i\alpha} pq + e^{i\alpha}\right), \\
y_{2,0} &= -\frac{1}{8}\, e^{-5i\alpha} \left(-e^{i\alpha} p + q\right)^2 \left(e^{i\alpha} p + q\right) \Big[7p^2 q^2 + 7e^{4i\alpha} p^2 q^2 + 2e^{i\alpha} pq - 2e^{3i\alpha} pq - \\
&\qquad - e^{2i\alpha} \left(p^4 + 12p^2 q^2 + q^4\right)\Big], \\
y_{3,0} &= \frac{1}{8}\, e^{-3i\alpha} \left(p + e^{i\alpha} q\right) \Big[-7p^3 q^3 - 7e^{6i\alpha} p^3 q^3 + e^{5i\alpha} p^2 q^2 \left(11p^2 + 3q^2\right) \\
&\qquad + e^{2i\alpha} pq \left(3p^4 + 9p^2 q^2 - 5q^4\right) + e^{4i\alpha} \left(-5p^5 q + 9p^3 q^3 + 3pq^5\right) + \\
&\qquad + e^{i\alpha} p^2 q^2 \left(3p^2 + 11q^2\right) + e^{3i\alpha} \left(p^6 - 15p^4 q^2 - 15p^2 q^4 + q^6\right)\Big]
\end{aligned} \tag{A.3}$$

with (2.5).

- SU(3) **sector of** SO(6, 2) **gauging**

The components of A_2 tensor can be represented by eight different fields $y_{i,-}(i = 1, 2, \ldots, 8)$ with degeneracies 3,3,4,12,12,4,6,12 respectively. This looks similar to the compact case(that is, same multiplicities and same number of fields) and they are given by

$$\begin{aligned}
{A_{2,7}}^{128} &= {A_{2,7}}^{348} = {A_{2,7}}^{568} \equiv y_{1,-} \\
{A_{2,8}}^{172} &= {A_{2,8}}^{374} = {A_{2,8}}^{576} \equiv y_{2,-} \\
{A_{2,7}}^{164} &= {A_{2,7}}^{135} = {A_{2,7}}^{263} = {A_{2,7}}^{254} \equiv y_{3,-} \\
{A_{2,1}}^{368} &= {A_{2,1}}^{458} = {A_{2,2}}^{358} = {A_{2,2}}^{486} = {A_{2,3}}^{186} = {A_{2,3}}^{285} = {A_{2,4}}^{185} = {A_{2,4}}^{268} = {A_{2,5}}^{148} \\
&= {A_{2,5}}^{238} = {A_{2,6}}^{138} = {A_{2,6}}^{284} \equiv y_{4,-} \\
{A_{2,1}}^{375} &= {A_{2,1}}^{467} = {A_{2,2}}^{367} = {A_{2,2}}^{457} = {A_{2,3}}^{157} = {A_{2,3}}^{276} = {A_{2,4}}^{176} = {A_{2,4}}^{275} = {A_{2,5}}^{173} \\
&= {A_{2,5}}^{247} = {A_{2,6}}^{147} = {A_{2,6}}^{237} \equiv y_{5,-} \\
{A_{2,8}}^{163} &= {A_{2,8}}^{154} = {A_{2,8}}^{253} = {A_{2,8}}^{246} \equiv y_{6,-} \\
{A_{2,1}}^{278} &= {A_{2,2}}^{187} = {A_{2,3}}^{478} = {A_{2,4}}^{387} = {A_{2,5}}^{678} = {A_{2,6}}^{587} \equiv y_{7,-} \\
{A_{2,1}}^{234} &= {A_{2,1}}^{256} = {A_{2,2}}^{143} = {A_{2,2}}^{165} = {A_{2,3}}^{124} = {A_{2,3}}^{456} = {A_{2,4}}^{132} = {A_{2,4}}^{365} = {A_{2,5}}^{126} \\
&= {A_{2,5}}^{346} = {A_{2,6}}^{152} = {A_{2,6}}^{354} \equiv y_{8,-}
\end{aligned} \tag{A.4}$$

where eight fields are

$$\begin{aligned}
y_{1,-} &= -\frac{1}{2}\, e^{-i(\alpha+4\phi)} \Big[e^{4i\phi} p^2 q r^4 + e^{i(3\alpha+4\phi)} p q^2 r^4 - e^{2i(\alpha+2\phi)} q \left(2p^2+q^2\right) r^4 - \\
&\qquad - e^{i(\alpha+4\phi)} p \left(p^2+2q^2\right) r^4 - 6e^{2i\phi} p^2 q r^2 t^2 - 6e^{i(3\alpha+2\phi)} p q^2 r^2 t^2 - \\
&\qquad - 2e^{2i(\alpha+\phi)} q \left(2p^2+q^2\right) r^2 t^2 - 2e^{i(\alpha+2\phi)} p \left(p^2+2q^2\right) r^2 t^2 + p^2 q t^4 + \\
&\qquad + e^{3i\alpha} p q^2 t^4 - e^{2i\alpha} q \left(2p^2+q^2\right) t^4 - e^{i\alpha} p \left(p^2+2q^2\right) t^4 \Big], \\
y_{2,-} &= -\frac{1}{2}\, e^{-i\alpha} \Big[p^2 q r^4 + e^{3i\alpha} p q^2 r^4 - e^{2i\alpha} q \left(2p^2+q^2\right) r^4 - e^{i\alpha} p \left(p^2+2q^2\right) r^4 - \\
&\qquad - 6e^{2i\phi} p^2 q r^2 t^2 - 6e^{i(3\alpha+2\phi)} p q^2 r^2 t^2 - 2e^{2i(\alpha+\phi)} q \left(2p^2+q^2\right) r^2 t^2 - \\
&\qquad - 2e^{i(\alpha+2\phi)} p \left(p^2+2q^2\right) r^2 t^2 + e^{4i\phi} p^2 q t^4 + e^{i(3\alpha+4\phi)} p q^2 t^4 - \\
&\qquad - e^{2i(\alpha+2\phi)} q \left(2p^2+q^2\right) t^4 - e^{i(\alpha+4\phi)} p \left(p^2+2q^2\right) t^4 \Big], \\
y_{3,-} &= -\frac{1}{2}\, e^{-3i\alpha} \left(p+e^{i\alpha} q\right) rt \Big[e^{4i\phi} p^2 r^2 - 4e^{i(\alpha+4\phi)} p q r^2 + e^{2i(\alpha+2\phi)} q^2 r^2 + p^2 t^2 - \\
&\qquad - 4e^{i\alpha} p q t^2 + e^{2i\alpha} q^2 t^2 - 3e^{2i\phi} p^2 \left(r^2+t^2\right) - \\
&\qquad - 3e^{2i(\alpha+\phi)} q^2 \left(r^2+t^2\right) \Big], \\
y_{4,-} &= -\frac{1}{2}\, e^{-i(2\alpha+3\phi)} rt \Big[e^{i(3\alpha+4\phi)} p^2 q r^2 + e^{4i\phi} p q^2 r^2 - e^{i(\alpha+4\phi)} q \left(2p^2+q^2\right) r^2 - \\
&\qquad - e^{2i(\alpha+2\phi)} p \left(p^2+2q^2\right) r^2 + e^{3i\alpha} p^2 q t^2 + p q^2 t^2 - \\
&\qquad - e^{i\alpha} q \left(2p^2+q^2\right) t^2 - e^{2i\alpha} p \left(p^2+2q^2\right) t^2 - \\
&\qquad - 3e^{i(3\alpha+2\phi)} p^2 q \left(r^2+t^2\right) - 3e^{2i\phi} p q^2 \left(r^2+t^2\right) - \\
&\qquad - e^{i(\alpha+2\phi)} q \left(2p^2+q^2\right) \left(r^2+t^2\right) - e^{2i(\alpha+\phi)} p \left(p^2+2q^2\right) \left(r^2+t^2\right) \Big], \\
y_{5,-} &= \frac{1}{2}\, e^{-i(2\alpha+\phi)} rt \Big[-e^{3i\alpha} p^2 q r^2 - p q^2 r^2 + e^{i\alpha} q \left(2p^2+q^2\right) r^2 + e^{2i\alpha} p \left(p^2+2q^2\right) r^2 -
\end{aligned}$$

$$
\begin{aligned}
&- e^{i(3\alpha+4\phi)}p^2qt^2 - e^{4i\phi}pq^2t^2 + e^{i(\alpha+4\phi)}q\left(2p^2+q^2\right)t^2 + \\
&+ e^{2i(\alpha+2\phi)}p\left(p^2+2q^2\right)t^2 + 3e^{i(3\alpha+2\phi)}p^2q\left(r^2+t^2\right) + \\
&+ 3e^{2i\phi}pq^2\left(r^2+t^2\right) + e^{i(\alpha+2\phi)}q\left(2p^2+q^2\right)\left(r^2+t^2\right) + \\
&+ e^{2i(\alpha+\phi)}p\left(p^2+2q^2\right)\left(r^2+t^2\right)\Big],
\end{aligned}
$$

$$
\begin{aligned}
y_{6,-} &= -\frac{1}{2}\,e^{-i\phi}\left(p+e^{i\alpha}q\right)rt\Big[-4e^{i\alpha}pq\left(r^2+e^{4i\phi}t^2\right) - e^{2i\alpha}q^2\,\times \\
&\quad \times\left(\left(-1+3e^{2i\phi}\right)r^2 - e^{2i\phi}\left(-3+e^{2i\phi}\right)t^2\right) + \\
&\quad + p^2\left(\left(1-3e^{2i\phi}\right)r^2 + e^{2i\phi}\left(-3+e^{2i\phi}\right)t^2\right)\Big], \\
y_{7,-} &= -\frac{1}{2}\,e^{-i(3\alpha+2\phi)}\left(e^{i\alpha}p+q\right)\Big[e^{2i\alpha}p^2r^2t^2 + e^{2i(\alpha+2\phi)}p^2r^2t^2 - 4e^{i\alpha}pqr^2t^2 - \\
&\quad - 4e^{i(\alpha+4\phi)}pqr^2t^2 + q^2r^2t^2 + e^{4i\phi}q^2r^2t^2 - \\
&\quad - e^{2i(\alpha+\phi)}p^2\left(r^4+4r^2t^2+t^4\right) - \\
&\quad - e^{2i\phi}q^2\left(r^4+4r^2t^2+t^4\right)\Big], \\
y_{8,-} &= \frac{1}{2}\,e^{-i(\alpha+2\phi)}\Big[-p^2qr^2t^2 - e^{4i\phi}p^2qr^2t^2 - e^{3i\alpha}pq^2r^2t^2 - e^{i(3\alpha+4\phi)}pq^2r^2t^2 + \\
&\quad + e^{2i\alpha}q\left(2p^2+q^2\right)r^2t^2 + e^{2i(\alpha+2\phi)}q\left(2p^2+q^2\right)r^2t^2 + \\
&\quad + e^{i\alpha}p\left(p^2+2q^2\right)r^2t^2 + e^{i(\alpha+4\phi)}p\left(p^2+2q^2\right)r^2t^2 + \\
&\quad + e^{2i\phi}p^2q\left(r^4+4r^2t^2+t^4\right) + e^{i(3\alpha+2\phi)}pq^2\left(r^4+4r^2t^2+t^4\right) + \\
&\quad + e^{i(\alpha+2\phi)}p\left(4q^2r^2t^2+p^2\left(r^4+t^4\right)\right) + \\
&\quad + e^{2i(\alpha+\phi)}q\left(4p^2r^2t^2+q^2\left(r^4+t^4\right)\right)\Big], \qquad \text{(A.5)}
\end{aligned}
$$

where we have (2.5) and (2.8).

SU(3) **sector of** CSO(6, 2) **gauging.** With $\xi = 0$, the components of A_2 tensor can be represented by eight different fields $y_{i,0}$ $(i = 1, 2, \ldots, 8)$ with degeneracies 3,3,4,12,12,4,6,12 respectively and given by (A.4) with the replacement $y_{i,-} \rightarrow y_{i,0}$ where their explicit expressions are given by

$$
\begin{aligned}
y_{1,0} &= -\frac{1}{4}e^{-i(\alpha+4\phi)}\left[3p^2q+3e^{3i\alpha}pq^2-e^{2i\alpha}q\left(2p^2+q^2\right)-e^{i\alpha}p\left(p^2+2q^2\right)\right]\left(-e^{2i\phi}r^2+t^2\right)^2, \\
y_{2,0} &= -\frac{1}{4}\,e^{-i\alpha}\left[3p^2q+3e^{3i\alpha}pq^2-e^{2i\alpha}q\left(2p^2+q^2\right)-e^{i\alpha}p\left(p^2+2q^2\right)\right]\left(r^2-e^{2i\phi}t^2\right)^2, \\
y_{3,0} &= -\frac{3}{4}\,e^{-3i\phi}\left(-1+e^{2i\phi}\right)\left(p-e^{i\alpha}q\right)^2\left(p+e^{i\alpha}q\right)rt\left(e^{2i\phi}r^2-t^2\right), \\
y_{4,0} &= -\frac{1}{4}\,e^{-i(2\alpha+3\phi)}\left(-1+e^{2i\phi}\right)\Big[3e^{3i\alpha}p^2q+3pq^2-e^{i\alpha}q\left(2p^2+q^2\right) - \\
&\quad - e^{2i\alpha}p\left(p^2+2q^2\right)\Big]rt\left(e^{2i\phi}r^2-t^2\right), \\
y_{5,0} &= -\frac{1}{4}\,e^{-i(2\alpha+\phi)}\left(-1+e^{2i\phi}\right)\Big[3e^{3i\alpha}p^2q+3pq^2-e^{i\alpha}q\left(2p^2+q^2\right) - \\
&\quad - e^{2i\alpha}p\left(p^2+2q^2\right)\Big]rt\left(-r^2+e^{2i\phi}t^2\right),
\end{aligned}
$$

$$
\begin{aligned}
y_{6,0} &= -\frac{3}{4}\, e^{-i\phi} \left(-1 + e^{2i\phi}\right) \left(p - e^{i\alpha} q\right)^2 \left(p + e^{i\alpha} q\right) rt \left(-r^2 + e^{2i\phi} t^2\right), \\
y_{7,0} &= -\frac{1}{4}\, e^{-i(3\alpha+2\phi)} \left(-e^{i\alpha} p + q\right)^2 \left(e^{i\alpha} p + q\right) \left[3r^2 t^2 + 3e^{4i\phi} r^2 t^2 - e^{2i\phi}\left(r^4 + 4r^2 t^2 + t^4\right)\right], \\
y_{8,0} &= \frac{1}{4}\, e^{-i(\alpha+2\phi)} \left(p - e^{i\alpha} q\right) \Big[-3pqr^2 t^2 + 3e^{2i\alpha} pqr^2 t^2 - 3e^{4i\phi} pqr^2 t^2 + 3e^{2i(\alpha+2\phi)} pqr^2 t^2 + \\
&\qquad + e^{i\alpha} r^2 t^2 + e^{i(\alpha+4\phi)} r^2 t^2 + e^{i(\alpha+2\phi)}\left(r^4 - 4r^2 t^2 + t^4\right) + \\
&\qquad + e^{2i\phi} pq \left(r^4 + 4r^2 t^2 + t^4\right) - e^{2i(\alpha+\phi)} pq \left(r^4 + 4r^2 t^2 + t^4\right)\Big]
\end{aligned} \tag{A.6}
$$

with (2.5) and (2.8).

$\mathrm{SO}(5)$ **sector of** $\mathrm{SO}(5,3)$ **gauging.** The components of A_2 tensor can be represented by four different fields with degeneracies $y_{i,-}(i = 1,2,3,4)$ 16,16,16,8 respectively and they look similar to the compact case (same multiplicities and same number of fields) and given by

$$
\begin{aligned}
A_{2,1}{}^{256} &= A_{2,1}{}^{278} = A_{2,2}{}^{165} = A_{2,2}{}^{187} = A_{2,3}{}^{456} = A_{2,3}{}^{478} = A_{2,4}{}^{365} = A_{2,4}{}^{387} = A_{2,5}{}^{126} \\
&= A_{2,5}{}^{346} = A_{2,6}{}^{152} = A_{2,6}{}^{354} = A_{2,7}{}^{128} = A_{2,7}{}^{348} = A_{2,8}{}^{172} = A_{2,8}{}^{374} \equiv y_{1,-} \\
A_{2,1}{}^{375} &= A_{2,1}{}^{368} = A_{2,2}{}^{486} = A_{2,2}{}^{457} = A_{2,3}{}^{186} = A_{2,3}{}^{157} = A_{2,4}{}^{275} = A_{2,4}{}^{268} = A_{2,5}{}^{173} \\
&= A_{2,5}{}^{247} = A_{2,6}{}^{138} = A_{2,6}{}^{284} = A_{2,7}{}^{135} = A_{2,7}{}^{254} = A_{2,8}{}^{163} = A_{2,8}{}^{246} \equiv y_{2,-} \\
A_{2,1}{}^{485} &= A_{2,1}{}^{476} = A_{2,2}{}^{385} = A_{2,2}{}^{376} = A_{2,3}{}^{267} = A_{2,3}{}^{258} = A_{2,4}{}^{167} = A_{2,4}{}^{158} = A_{2,5}{}^{184} \\
&= A_{2,5}{}^{283} = A_{2,6}{}^{174} = A_{2,6}{}^{273} = A_{2,7}{}^{146} = A_{2,7}{}^{236} = A_{2,8}{}^{145} = A_{2,8}{}^{235} \equiv y_{3,-} \\
A_{2,1}{}^{234} &= A_{2,2}{}^{143} = A_{2,3}{}^{124} = A_{2,4}{}^{132} = A_{2,5}{}^{678} = A_{2,6}{}^{587} = A_{2,7}{}^{568} = A_{2,8}{}^{576} \equiv y_{4,-}
\end{aligned} \tag{A.7}
$$

where they have explicit simple form

$$
\begin{aligned}
y_{1,-} &= \frac{1}{8\sqrt{uvw}} \left(1 + u^2 v^2 + u^2 w^2 - v^2 w^2\right), \\
y_{2,-} &= \frac{1}{8\sqrt{uvw}} \left(1 + u^2 v^2 - u^2 w^2 + v^2 w^2\right), \\
y_{3,-} &= \frac{1}{8\sqrt{uvw}} \left(1 - u^2 v^2 + u^2 w^2 + v^2 w^2\right), \\
y_{4,-} &= \frac{1}{8\sqrt{uvw}} \left(3 + u^2 v^2 + u^2 w^2 + v^2 w^2\right)
\end{aligned} \tag{A.8}
$$

together with (2.13).

$\mathrm{SO}(5)$ **sector of** $\mathrm{CSO}(5,3)$ **gauging.** With $\xi = 0$, the components of A_2 tensor can be represented by two different fields $y_{i,0}$ $(i = 1,2)$ with degeneracies 48,8 respectively and given by

$$
\begin{aligned}
A_{2,1}{}^{256} &= A_{2,1}{}^{278} = A_{2,2}{}^{165} = A_{2,2}{}^{187} = A_{2,3}{}^{456} = A_{2,3}{}^{478} = A_{2,4}{}^{365} = A_{2,4}{}^{387} = A_{2,5}{}^{126} \\
&= A_{2,5}{}^{346} = A_{2,6}{}^{152} = A_{2,6}{}^{354} = A_{2,7}{}^{128} = A_{2,7}{}^{348} = A_{2,8}{}^{172} = A_{2,8}{}^{374} = A_{2,1}{}^{375} \\
&= A_{2,1}{}^{368} = A_{2,2}{}^{486} = A_{2,2}{}^{457} = A_{2,3}{}^{186} = A_{2,3}{}^{157} = A_{2,4}{}^{275} = A_{2,4}{}^{268} = A_{2,5}{}^{173}
\end{aligned}
$$

$$
\begin{aligned}
&= A_{2,5}{}^{247} = A_{2,6}{}^{138} = A_{2,6}{}^{284} = A_{2,7}{}^{135} = A_{2,7}{}^{254} = A_{2,8}{}^{163} = A_{2,8}{}^{246} = A_{2,1}{}^{485} \\
&= A_{2,1}{}^{476} = A_{2,2}{}^{385} = A_{2,2}{}^{376} = A_{2,3}{}^{267} = A_{2,3}{}^{258} = A_{2,4}{}^{167} = A_{2,4}{}^{158} = A_{2,5}{}^{184} \\
&= A_{2,5}{}^{283} = A_{2,6}{}^{174} = A_{2,6}{}^{273} = A_{2,7}{}^{146} = A_{2,7}{}^{236} = A_{2,8}{}^{145} = A_{2,8}{}^{235} \equiv y_{1,0} \\
A_{2,1}{}^{234} &= A_{2,2}{}^{143} = A_{2,3}{}^{124} = A_{2,4}{}^{132} = A_{2,5}{}^{678} = A_{2,6}{}^{587} = A_{2,7}{}^{568} = A_{2,8}{}^{576} \equiv y_{2,0} \qquad \text{(A.9)}
\end{aligned}
$$

where we have

$$
y_{1,0} = \frac{1}{8\sqrt{uvw}}, \qquad y_{2,0} = \frac{3}{8\sqrt{uvw}} \qquad \text{(A.10)}
$$

with (2.13).

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